5. **Recall**:

$$\text{Recall} = \frac{40}{40+10} = \frac{40}{50} = 0.8 \text{ or } 80\%$$

6. **F1 Score**:

$$\text{F1 Score} = 2 \times \frac{0.888 \times 0.8}{0.888 + 0.8} = 0.842$$

So, the **F1 Score** for this model is **0.842**, indicating a good balance between precision and recall.

Receiver Operating Characteristic (ROC) Curve and AUC

The **Receiver Operating Characteristic (ROC)** curve and **Area Under the Curve (AUC)** are essential tools for evaluating the performance of binary classification models. They help assess the trade-offs between different types of errors (false positives and false negatives) and visualize the model's performance across various thresholds.

ROC Curve (Receiver Operating Characteristic Curve)

The **ROC curve** is a graphical representation that illustrates the diagnostic ability of a binary classifier as its discrimination threshold is varied. It plots the **True Positive Rate (TPR)** against the **False Positive Rate (FPR)** at different threshold settings.

- **True Positive Rate (TPR)**, also known as **Recall** or **Sensitivity**:

$$\text{TPR} = \frac{TP}{TP + FN}$$

  - **TP** = True Positives (correctly predicted positive cases)
  - **FN** = False Negatives (incorrectly predicted negative cases)
- **False Positive Rate (FPR)**:

$$\text{FPR} = \frac{FP}{FP + TN}$$

  - **FP** = False Positives (incorrectly predicted positive cases)
  - **TN** = True Negatives (correctly predicted negative cases)

Interpretation of the ROC Curve

7. The **x-axis** represents the **False Positive Rate (FPR)**, which shows how often the model incorrectly classifies negative cases as positive.
8. The **y-axis** represents the **True Positive Rate (TPR)**, which shows how often the model correctly classifies positive cases as positive.

A **perfect classifier** would have a point at the top-left corner of the plot, where the FPR is 0 and the TPR is 1, indicating 100% sensitivity (no false negatives) and 100% specificity (no false positives).

A **random classifier** would plot a diagonal line from the bottom-left to the top-right corner of the plot (FPR = TPR), showing that the model performs no better than random guessing.

The **steeper the curve** toward the top-left corner, the better the model is at distinguishing between positive and negative cases.

Area Under the ROC Curve (AUC)

AUC stands for **Area Under the Curve** and is a single scalar value that summarizes the overall performance of a binary classifier across all classification thresholds.

6. **AUC** is the area under the ROC curve.
7. A **higher AUC** indicates a better-performing model. The value of AUC ranges from **0 to 1**:
   a. **AUC = 1**: The model is perfect, classifying all positives and negatives correctly.
   b. **AUC = 0.5**: The model performs no better than random guessing.
   c. **AUC < 0.5**: The model is performing worse than random, indicating that the model is systematically making wrong predictions.

Interpretation of AUC

- **AUC = 1**: A perfect model with 100% sensitivity and specificity.
- **AUC = 0.9 - 0.99**: Excellent model.
- **AUC = 0.7 - 0.9**: Good model.
- **AUC = 0.6 - 0.7**: Fair model.
- **AUC = 0.5**: The model has no discrimination power; it is like a random classifier.
- **AUC < 0.5**: The model is worse than random guessing (this can indicate that the model is systematically misclassifying cases).

ROC Curve Example

Imagine a binary classifier for detecting whether an email is **spam** or **not spam**. We would calculate the **TPR** and **FPR** at various threshold values. For example:

- If the threshold for classification is very low, the model may predict most emails as spam (leading to high recall but also many false positives).
- If the threshold is very high, the model may only predict very confident cases as spam, leading to fewer false positives but also more false negatives.

As we vary the threshold, we plot the corresponding TPR and FPR values on the ROC curve. The ideal model would have a high TPR and a low FPR, creating a curve that steers toward the top-left corner.

Advantages of Using ROC and AUC

6. **Threshold Independence**: The ROC curve evaluates the classifier's performance across all possible classification thresholds, providing a comprehensive view of the model's behavior.
7. **Class Imbalance**: Unlike metrics like accuracy, which can be misleading when dealing with imbalanced datasets, the ROC curve and AUC are not affected by class imbalance. This makes them valuable in scenarios where one class is much less frequent than the other (e.g., fraud detection, disease diagnosis).
8. **Comprehensive Model Comparison**: ROC and AUC allow you to compare multiple models or classifiers by looking at how their curves compare. The model with the highest AUC is generally the best at distinguishing between the classes.

Median Absolute Deviation (MAD)

Median Absolute Deviation (MAD) is a robust statistical measure of the spread or variability of a dataset. It is used to quantify the dispersion of data, similar to standard deviation, but it is less sensitive to outliers, making it a more robust measure in datasets with extreme values or outliers.

Formula for MAD

The **Median Absolute Deviation** is defined as the median of the absolute deviations from the median of the dataset. The steps to calculate MAD are:

3. **Find the Median** of the data points.
4. **Calculate the Absolute Deviation** for each data point by subtracting the median from each data point and taking the absolute value of the result.

$$|x_i - \mathrm{Median}|$$

Where: