Then compression in the aluminium cylinder

$$= (\delta - 0.20)\,\text{mm}$$

$\therefore$ Strain in steel, $e_S = \dfrac{\delta}{500}$

Stress in aluminium, $e_a = \dfrac{\delta - 0.20}{500 - 0.20} = \dfrac{\delta - 0.20}{499.8}$

Strain in steel, $\sigma_S = E_S \times e_S$

$$= 210 \times 10^3 \times \frac{\delta}{500} = 420\delta\ \text{N/mm}^2$$

Stress in aluminium, $\sigma_a = E_a \times e_a$

$$= 70 \times 10^3 \times \frac{\delta - 0.20}{499.8} = 140.06\ (\delta - 0.20)$$

Total load = Load on steel + Load on aluminium

$$400 \times 10^3 = \sigma_S A_S + \sigma_a A_a$$

$$= 420\delta \times 2827.43 \times 140.06(\delta - 0.20)$$

$$400 \times 10^3 = 1187520.6 + 140.06\delta - 28.01$$

$$400 \times 10^3 \times 28.01 = 1187660.7\delta$$

$$\delta = 0.337\ \text{mm}$$

$\therefore$ $\sigma_S = 420\ \delta = 420 \times 0.337 = 141.54\ \text{N/mm}^2$

and, $\sigma_a = 140.06\ (\delta - 0.20) = 140.06\ (0.337 - 0.20)$

$$= 19.19\ \text{N/mm}^2$$

7.29 FLEXURE LOADING

7.29.1 Beam

A structural member which carries lateral or transverse force (forces at right angles to the axis of the member) is termed as a beam or joist. Generally a beam is of moderate size and is made up of one piece. If the size is large and the beam is made of many parts joined together, than it is called girder.

7.29.2 Type of Beam

Depending upon the end conditions the various types of beams are:

1. Cantilever beam
2. Simply supported beam
3. Overhanging beam
4. Fixed beam, and
5. Continuous beam.

1. Cantilever Beam: A beam which is fixed at one end and free at the other end, is known as cantilever beam. (Fig. 31)

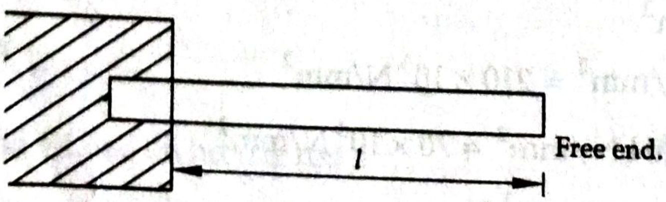


Fig. 31

where l = Length of Cantilever Beam.

2. Simply supported Beam: A beam supported or resting freely on the supported at its both ends, is known as simply or freely supported beam (Fig. 32). The supported may be in the form of walls or columns.

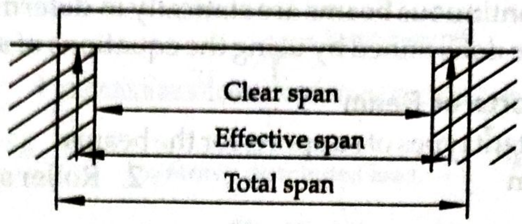


Fig. 32

The whole length of the beam is known as its total span. The clear horizontal distance between the walls is called the clear span of the beam. The horizontal distance between the centres of the end bearing is called the effective span of the beam.

3. Overhanging Beam: A beam in which its end portion (or portion) is extended beyond the support, is known as overhanging beam (Fig. 33). A beam may be overhanging on one side or on both sides. (As shown in Fig. 33 a, b & c).

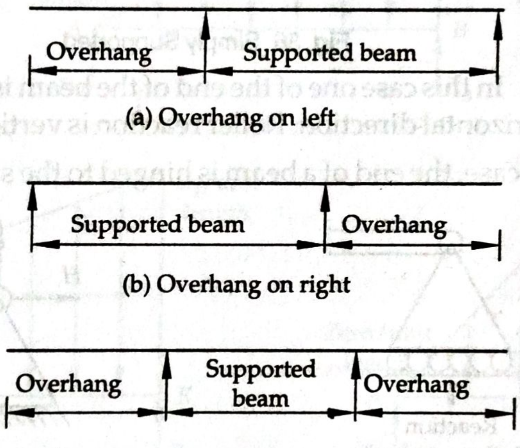


Fig. 33

4. Fixed Beam: A beam whose both ends are fixed or built-in walls, is known as fixed beam. As shown in Fig. 34.

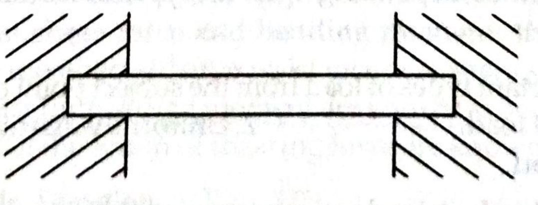


Fig. 34

5. Continuous Beam: A Beam supported on more than two supports is known as continuous beam (Fig. 35). The extreme left and right supports are called and supports and all the remaining supports are known as intermediate supports.

(Continuous Beam)

Fig. 35

Cantilever, simply supported beams and overhanging beams are statically determinates beams.

In such beams supports reactions can be determined by using the equations of static equilibrium. The equations of static equilibrium are

$$\Sigma H = 0,\ \Sigma V = 0\ \Sigma M = 0$$

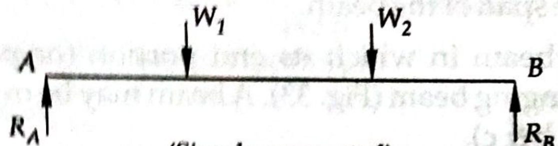
Fixed end beams and continuous beams are statically in determinates beams. In such beams the supports reactions can not be determined by using the equations of static equilibrium.

7.29.3 Types of end supports of Beam

The following are the important types of supports for the beams:

1. Simply supported beam
2. Roller supported beams, and
3. Hinged beams.

1. Simply supported beams: A beam which rests freely on supports at its both ends is called simply supported or freely supported or knife edge supported beam. The Supports may be in the form of walls or columns. In such a case, the reaction is always vertical as shown in Fig. 36.

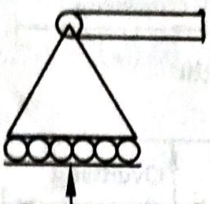


(Simply supported)

Fig. 36 Simply Supported

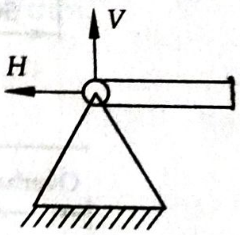
2. Roller supported beams: In this case one of the end of the beam is supported on rollers in order to permit free movement in horizontal direction. Roller reaction is vertical as shown in Fig. 37.

3. Hinged Beam: In such a case, the end of a beam is hinged to the supported as shown in Fig. 38.



Reaction
Roller support

Fig. 37



Hinged Beam

Fig. 38

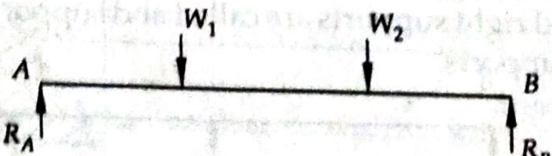
The structural members supported on hinged supported can rotate about hinge but cannot move side ways. Thus the positions is fixed in this case. The reaction on a hinged supported may be either horizontal or vertical or inclined depending upon the types of loading on the structure.

8.29.4 Types of Load

The following are the important types of load from the subject point of view.

1. Concentrated or point load.
2. Uniformly distributed load, and
3. Uniformly varying load.

1. Concentrated or point load: A load acting at a point on the beam is known as a concentrated or a point load as shown in Fig. 39.



(Point Load)

Fig. 39

2. Uniformly distributed load: A load which is spread uniformly over the entire span or small portion of the beam is known as uniformly distributed load (U.D.L) as shown in Fig. 40.

This load is assumed to act at the C.G. of the load for all types of calculations.

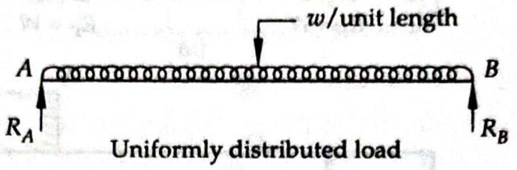


Fig. 40

3 Uniformly varying load: A load which is spread over a beam, in such a manner that the rate of loading varies from one point to another, is known as a uniformly varying load as shown in Fig. 41.

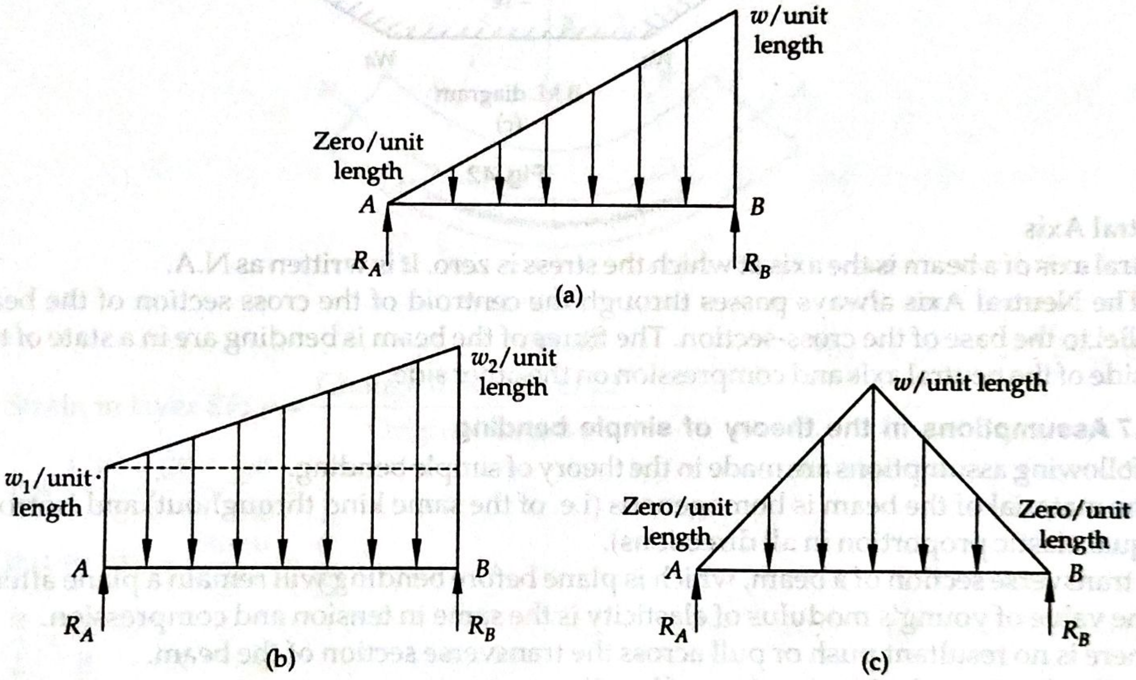


Fig. 41

7.29.5 Bending stresses

When some external load acts on a beam, the shear force and bending moments are set up at all section of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The material of the member will offer resistance or stresses against these deformations.

The stresses produced to resist the bending moment are known as bending stresses.

The stresses produced to resist the action of shearing force are known as shearing stresses.

7.29.6 Pure Bending or Simple Bending

The bending of the beam not accompanied by any shear force is termed as pure bending or simple bending.

Fig. 42 shows a beam *ABCD* with equal over hangs and supported at *B* and *C*. Let a point load *W* be applied at each end of the beam. S.F. and B.M. diagrams are drawn for the given loading.

From these diagrams, it is clear that there is no shear force between *B* and *C* but the B.M. between *B* and *C* is constant.

This means that between *B* and *C*, the beam is subjected to a constant bending moment only. This condition of the beam between *B* and *C* is known as pure bending or simple bending.

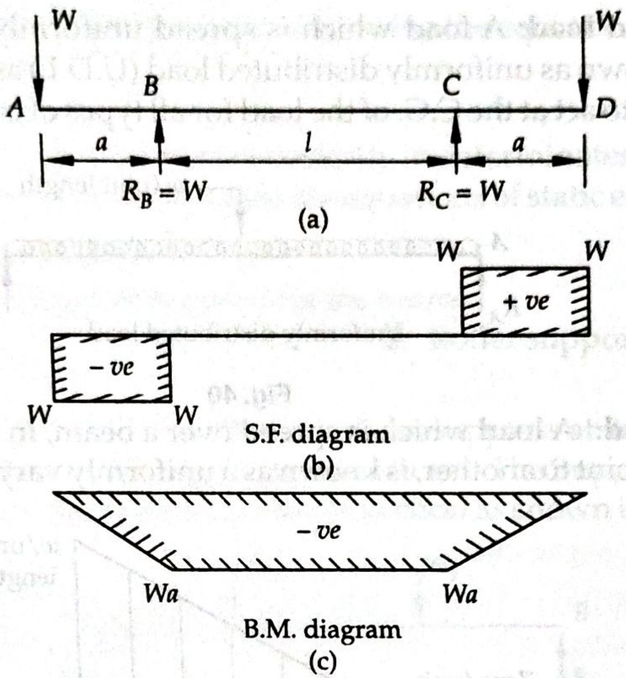


Fig. 42

Neutral Axis

Neutral axis of a beam is the axis at which the stress is zero. It is written as N.A.

The Neutral Axis always passes through the centroid of the cross section of the beam and is parallel to the base of the cross-section. The fibres of the beam is bending are in a state of tension on one side of the neutral axis and compression on the other side.

7.29.7 Assumptions in the theory of simple bending

The following assumptions are made in the theory of simple bending.

1. The material of the beam is homogenous (i.e. of the same kind throughout) and isotropic (i.e. of equal elastic proportion in all directions).
2. A transverse section of a beam, which is plane before bending will remain a plane after bending.
3. The value of young's modulus of elasticity is the same in tension and compression.
4. There is no resultant push or pull across the transverse section of the beam.
5. The loads are applied in the plane of bending.
6. The beam material is stressed with in its elastic limit and thus obeys Hooke's law.
7. The radius of curvature of the beam before bending is very large as compared to the transverse dimensions of the beam.
8. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.