7.14 EFFECT OF STRESS AS LONGITUDINAL AND LATERAL STRAIN

(Refer Fig. 8)

Strain due to σ_1 :

In x direction i.e. longitudinal strain $= \frac{\sigma_1}{E}$ $\left[\because E = \frac{\sigma}{e}, \text{ so longitudinal strain } = \frac{\sigma}{E}\right]$

In y direction i.e. lateral strain $= -v\frac{\sigma_1}{E}$

– ve sign shows that it is reverse, because if longitudinal strain is tensile then lateral strain is compressive. $\left[v = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}, \text{ lateral strain} = v \times \text{Longitudinal strain}\right]$

Strain due to σ_2 :

In x direction i.e. lateral strain $= -v\frac{\sigma_2}{E}$

In y direction i.e. longitudinal strain $= \frac{\sigma_2}{E}$

Now strain in x direction due to stresses (σ_1 and σ_2)

$$\frac{\sigma_1}{E} - v\frac{\sigma_2}{E}$$

and strain in y direction due to stresses (σ_1 and σ_2)

$$\frac{\sigma_2}{E} - v\frac{\sigma_1}{E}$$

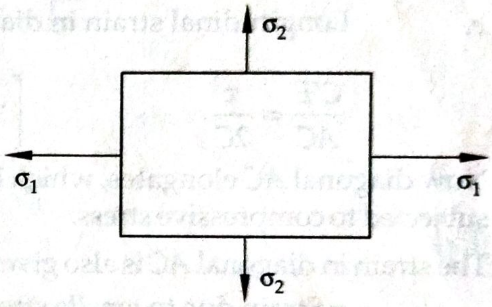

Fig. 8

7.15 RELATION BETWEEN THREE ELASTIC CONSTANTS i.e. K, E, C

E = Young's modulus of elasticity

C = Modulus of rigidity or shear modulus

K = Bulk modulus of elasticity

The relation between E, C and K helps us in determining the strains produced in a given material under the action of a given loading system.

7.15.1 Relation between E and C

A cube $ABCD$ is distorted by a shear stress τ, and the shape $ABCD$ is changed to $ABC'D'$.

Draw a perpendicular CE to AC' as shown in Fig. 9. As the deformation is very small, angle $AC'C$ may be taken as 45°.

Shearing stress is induced at the faces DC and AB

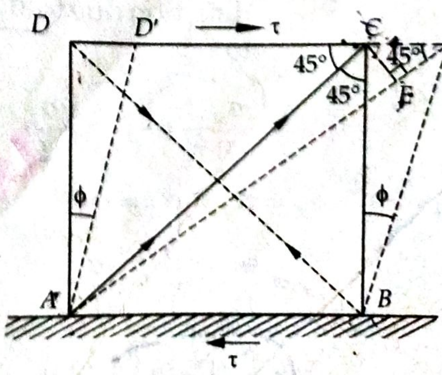

Fig. 9

$$\text{Shear strain } \phi = \frac{CC'}{BC} \quad \text{...(i)}$$

$$\text{Also shear strain } = \frac{\tau}{C} \quad \text{...(ii)}$$

In right angled triangle CEC'

$$\frac{C'E}{CC'} = \cos 45°$$

or $$CC' = \frac{C'E}{\cos 45°} = \sqrt{2}\,C'E \quad \left(\because \cos 45° = \frac{1}{\sqrt{2}}\right)$$

In right angled triangle ABC

$$\frac{BC}{AC} = \cos 45°$$

$$BC = AC \cos 45° = \frac{AC}{\sqrt{2}}$$

Putting the values of CC' and BC in Eq. (i) and compare with Eq. (ii), we get

$$\frac{\tau}{C} = \frac{\sqrt{2}\, C'E \sqrt{2}}{AC} = \frac{2C'E}{AC} \qquad \frac{\tau}{C} = \frac{2C'E}{AC} \qquad \text{...(iii)}$$

Since AC is very nearly equal to AE

∴ $C'E$ is the increase in length of diagonal AC

∴ Longitudinal strain in diagonal $AC = \frac{C'E}{AC}$

$$\boxed{\frac{C'E}{AC} = \frac{\tau}{2C}} \qquad \left[\because \frac{C'E}{AC} = \frac{\tau}{2C} \text{ from Eq.(iii)}\right] \qquad \text{...(iv)}$$

Now diagonal AC elongates, which indicates that it is subjected to tensile stress and diagonal BD is subjected to compressive stress.

The strain in diagonal AC is also given by

= Strain due to tensile stress in AC – Strain due to compressive stress in BD

$$\frac{C'E}{AC} = \frac{\tau}{E} - \left(-v\frac{\tau}{E}\right) = \frac{\tau}{E}(1+v) \qquad \text{...(v)}$$

From Eq. (iv) and (v)

$$\frac{\tau}{2C} = \frac{\tau}{E}(1+v)$$

$$\boxed{E = 2C(1+v)} \qquad \text{...(vi)}$$

7.15.2 Relation between E and K

Consider a cubical element subjected to volumetric stress σ which acts simultaneously along the mutually perpendicular x, y and z direction as shown in Fig. 10.

The resultant strains along the three directions can be worked out by taking the effect of individual stresses.

Strain in the x direction,

e_x = Strain in x direction due to σ_x – Strain in x direction due to σ_y – Strain in x direction due to σ_z.

$$= \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$$

But $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$\therefore \quad e_x = \frac{\sigma}{E} - v\frac{\sigma}{E} - v\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1-2v)$$

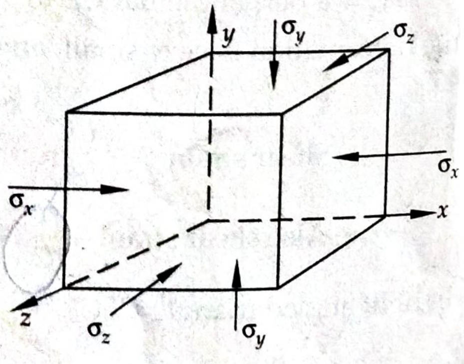

Fig. 10

Similarly strain in the y direction

$$e_y = \frac{\sigma}{E}(1 - 2v)$$

Strain in z direction

$$e_z = \frac{\sigma}{E}(1 - 2v)$$

Volumetric strain $e_v = e_x + e_y + e_z = \frac{3\sigma}{E}(1 - 2v)$

Bulk modulus

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

Normal Str

$$= \frac{\sigma}{\frac{3\sigma}{E}(1 - 2v)}$$

$$\frac{\sigma}{3\sigma} \quad E$$

$$K = \frac{E}{3(1 - 2v)}$$

$$3K(1-2v)$$

$$\boxed{E = 3K(1 - 2v)} \qquad \text{...(vii)}$$

7.15.3 Relation between E, C and K

Taking Eq. (vi) and (vii)

$$E = 2C(1 + v) \qquad\qquad E = 3K(1 - 2v)$$

$$\frac{E}{2C} = 1 + v \qquad\qquad \frac{E}{3K} = 1 - 2v$$

$$v = \frac{E}{2C} - 1 \qquad\qquad 2v = 1 - \frac{E}{3K}$$

$$v = \frac{E - 2C}{2C} \qquad\qquad v = \frac{3K - E}{6K}$$

$$\frac{E - 2C}{2C} = \frac{3K - E}{6K}$$

Cross multiplying, we get

$$(E - 2C)\,6K = 2C(3K - E)$$

$$6KE - 12CK = 6CK - 2CE$$

$$6KE + 2CE = 18CK$$

$$3KE + CE = 9CK$$

$$(3K + C)E = 9CK$$

$$E = \frac{9CK}{3K + C}$$

$$\boxed{E = \frac{9CK}{3K + C}}$$