## 7.2 STRESS

Whenever a body is subjected to an external force, it tends to undergo deformation (i.e. change in shape or dimension). Due to cohesion between the molecules, the body resists deformation.

"The force of resistance per unit area offered by a body against the deformation is called stress."

The external force acting on the body is called the load. Stress is usually denoted by $\sigma$ (sigma).

Mathematically, stress is written as

$$\sigma = P/A$$

$\sigma = P/A$ — cross section area.

where $\sigma$ = Stress

$P$ = External force or load and

$A$ = Cross-sectional area

The stress is expressed in $N/m^2$ or $N/mm^2$. In S.I. Units $N/m^2$ is called Pascal abbreviated as *Pa.*

$$1\ MN/m^2 = 1\ MPa = 1 \times 10^6\ N/m^2 = 1\ N/mm^2$$

$$1\ GPa = 1000\ MPa = 1000\ N/mm^2 = 1\ kN/mm^2$$

## 7.3 TYPES OF STRESS

(i) *Tensile stress:* When two equal and opposite pulls are applied to a member, the stress induced in a member is known as tensile stress.

Let an axial pull $P$ acting on a member of cross-sectional area $A$.

Then tensile stress ($\sigma_t$) is given by

$$\sigma_t = P/A$$

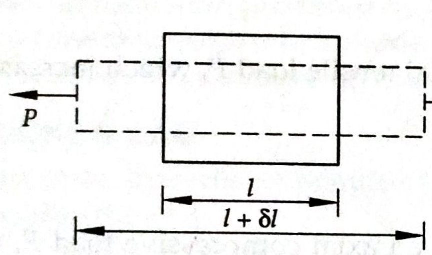


Fig. 2

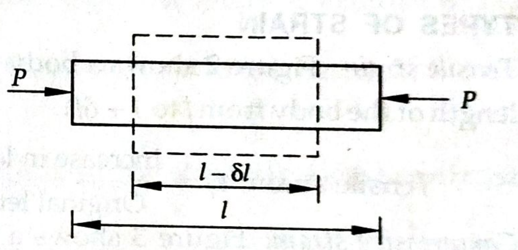


Fig. 3

(ii) *Compressive stress:* When two equal and opposite pushes are applied to a member, the stress induced in a member is known as compressive stress.

Let an axial push $P$ acting on a member of cross-sectional area $A$.

The compressive stress ($\sigma_c$) is given by

$$\sigma_c = P/A$$

(iii) *Shear stress:* When two equal and opposite forces are acting tangentially to the cross-section, the stress induced in a member is known as shear stress.

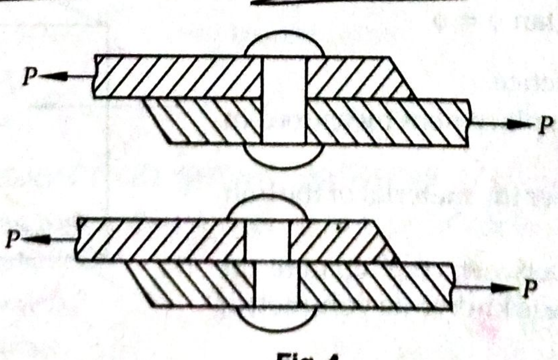


Fig. 4

Due to these two equal and opposite forces, the member tends to shear off across the section.

Figure 4 shows the failure of a rivet due to shear. The shear stress is also known as tangential stress.

Shear stress is denoted by ($\tau$)

Consider a block $ABCD$ with bottom face $AB$ fixed to the surface as shown in Fig. 5. When force $P$ is applied tangentially, the block takes the shape $ABC'D'$.

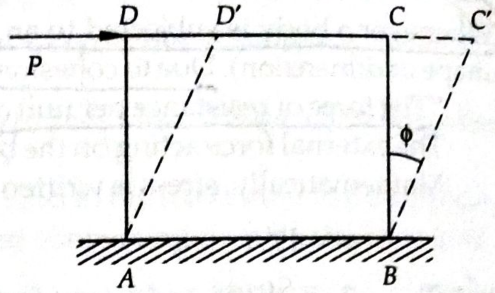


Fig. 5

If $A$ = Shear area, ($AB$ × width of block)

Then, Shear stress, $\tau = \dfrac{\text{Shear force}}{\text{Shear Area}} = \dfrac{P}{A}$

$\tau = PA$

## 7.4 STRAIN

It is a measure of the deformation or change in shape suffered by the loaded body.

The ratio of change of dimension of the body to the original dimension is known as strain. It is denoted by $e$.

$$\therefore \quad \text{Strain, } e = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

$e = \dfrac{\text{change in dim}}{\text{original}}$

Strain is a dimensionless quantity.

## 7.5 TYPES OF STRAIN

1. *Tensile strain:* Figure 2 shows a body subjected to an axial tensile load $P$, which increases the length of the body from $l$ to $l + \delta l$.

$$\therefore \quad \text{Tensile strain, } e_t = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\delta l}{l}$$

2. *Compressive strain:* Figure 3 shows a body subjected to an axial compressive load $P$, which decreases the length of the body from $l$ to $l - \delta l$.

$$\therefore \quad \text{Compressive strain, } e_c = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{\delta l}{l}$$

3. *Shear strain:* It is a measure of the angle through which a body is distorted under the action of shear forces.

   In Fig. 5 the angular displacement is given by $\phi$ which measure the shear strain, $\phi$ being in radians.

   $$\text{Shear strain, } e_s = \frac{CC'}{BC} = \tan\phi = \phi$$

   $\phi$ being very small in practice.

   So, shear strain is an angular displacement measured in radian.

   When a bolt is turned by a spanner the material of the bolt is in a state of shear strain.

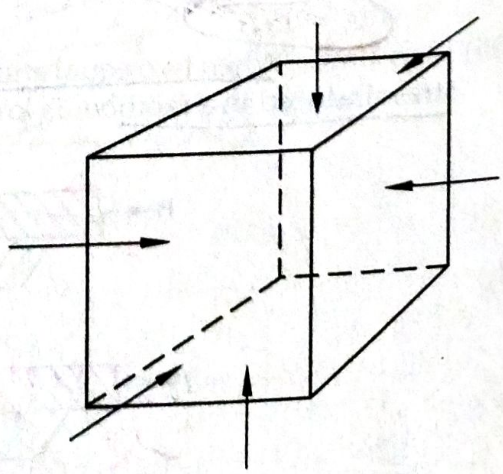

Fig. 6

4. *Volumetric strain:* The ratio between the change in volume and the original volume is known as volumetric strain or bulk strain.

$\therefore$ Volumetric strain, $e_V = \dfrac{\text{Change in volume}}{\text{Original volume}} = \dfrac{\delta V}{V}$

Let a cube is subjected to uniform compressive force on all its faces as shown in Fig. 6. Let the side of the cube change from $l$ to $l - \delta l$ after the application of compressive force.

Then, change in volume $= l^3 - (l - \delta l)^3$

$$= l^3 - (l^3 - 3l^2\delta l + 3l\delta l^2 - \delta l^3)$$

$$= 3l^2\delta l \quad \text{(neglecting higher powers of small quantity } \delta l\text{)}$$

Then, volumetric strain

$$e_V = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= \frac{3l^2\delta l}{l^3} = \frac{3\delta l}{l}$$

Hence, volumetric strain of a cube is three times the linear strain of one of its edges.

## 7.6 ELASTICITY

A material is said to be perfectly elastic if the deformation produced in it by external forces, completely disappears on the removal of external forces.

This property by virtue of which certain materials return back to their original position after the removal of the external load or forces is called elasticity.

If the deformation produced by external forces do not disappear after the external forces are removed, the material is called plastic.

A material remains elastic up to a certain stress called the elastic limit of the material.

## 7.7 HOOKE'S LAW

This law states that when a material is loaded within elastic limit, the stress is proportional to strain produced by the stress.

$\therefore$ Stress $\propto$ Strain

Stress = a constant × strain

$$\frac{\text{Stress}}{\text{Strain}} = \text{a constant}$$

This constant is known as modulus of elasticity.

## 7.8 YOUNG'S MODULUS OF ELASTICITY (E)

The ratio between tensile stress and tensile strain or between compressive stress and compressive strain is called Young's Modulus of Elasticity. This is denoted by E.

$$\therefore \quad E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \text{ or } \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

or $$E = \frac{\sigma_t}{e_t} \text{ or } \frac{\sigma_c}{e_c} \text{ or } \frac{\sigma}{e}$$

## 7.9 MODULUS OF RIGIDITY OR SHEAR MODULUS (G)

The ratio of shear stress to the shear strain is known as shear modulus of elasticity or modulus of rigidity. It is denoted by C or G.

$$C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi} \qquad = \tau/\phi$$

where $\tau$ is shear stress and $\phi$ is shear strain.

## 7.10 BULK MODULUS OF ELASTICITY (K)

The ratio of normal stress to the volumetric strain is called bulk modulus of elasticity. It is denoted by *K*.

Mathematically bulk modulus is given by

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_V}$$

$$\frac{\sigma}{e_v}$$

## 7.11 LONGITUDINAL STRAIN

When a body is subjected to an axial tensile load, there is an axial deformation in the length of the body.

The ratio of axial deformation to the original length of the body is known as longitudinal or linear or primary strain. Refer to Fig. 7.

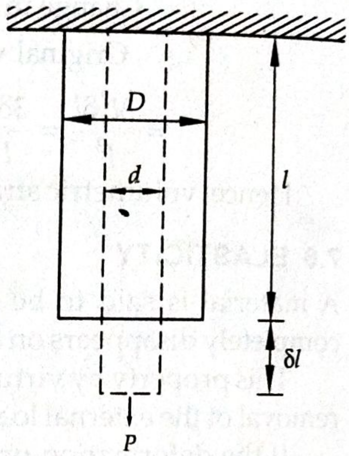

Fig. 7

$$\text{Longitudinal strain} = \frac{\delta l}{l}$$

Let $l$ = Original length of the rod

$P$ = Tensile force acting on the rod

$\delta l$ = Increase in length of the rod in the direction of $P$

Then, $\text{Longitudinal strain} = \frac{\delta l}{l}$

## 7.12 LATERAL STRAIN

When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body but at the same time there is a change in other dimensions of the body at right angles to the line of action of the applied load. Thus, the body is having axial deformation and also deformation at right angles to the line of action of the applied load.

The strain at right angles to the direction of applied load is known as lateral or transverse or secondary strain. Refer to Fig. 7.

Let $D$ = Original diameter of the rod

$d$ = Reduced diameter after elongation

$P$ = Tensile force acting on the rod

(3)

Then, $\text{Lateral strain} = \frac{D-d}{D}$

If longitudinal strain is tensile, the lateral strains will be compressive and if longitudinal strain is compressive then lateral strains will be tensile.

## 7.13 POISSON'S RATIO

The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

It is denoted by $v$ or $\frac{1}{m}$ (μ)

$$v = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$v = \frac{\text{later}}{\text{lo st}}$$

## 7.14 EFFECT OF STRESS AS LONGITUDINAL AND LATERAL STRAIN

(Refer Fig. 8)

**Strain due to $\sigma_1$ :**

In $x$ direction i.e. longitudinal strain $= \dfrac{\sigma_1}{E}$ $\left[\because E = \dfrac{\sigma}{e}, \text{ so longitudinal strain } = \dfrac{\sigma}{E}\right]$

In $y$ direction i.e. lateral strain $= -v\dfrac{\sigma_1}{E}$

– ve sign shows that it is reverse, because if longitudinal strain is tensile then lateral strain is compressive. $\left[v = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}, \text{ lateral strain} = v \times \text{Longitudinal strain}\right]$

**Strain due to $\sigma_2$ :**

In $x$ direction i.e. lateral strain $= -v\dfrac{\sigma_2}{E}$

In $y$ direction i.e. longitudinal strain $= \dfrac{\sigma_2}{E}$

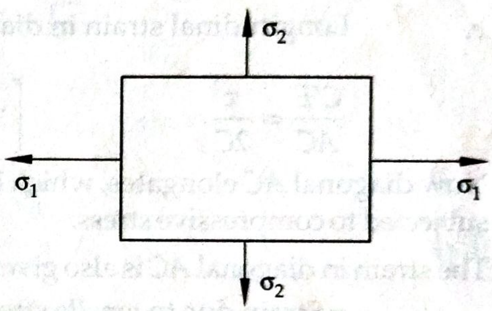

Fig. 8

Now strain in $x$ direction due to stresses ($\sigma_1$ and $\sigma_2$)

$$\frac{\sigma_1}{E} - v\frac{\sigma_2}{E}$$

and strain in $y$ direction due to stresses ($\sigma_1$ and $\sigma_2$)

$$\frac{\sigma_2}{E} - v\frac{\sigma_1}{E}$$

## 7.15 RELATION BETWEEN THREE ELASTIC CONSTANTS i.e. K, E, C

$E$ = Young's modulus of elasticity

$C$ = Modulus of rigidity or shear modulus

$K$ = Bulk modulus of elasticity

The relation between $E$, $C$ and $K$ helps us in determining the strains produced in a given material under the action of a given loading system.

### 7.15.1 Relation between E and C

A cube $ABCD$ is distorted by a shear stress $\tau$, and the shape $ABCD$ is changed to $ABC'D'$.

Draw a perpendicular $CE$ to $AC'$ as shown in Fig. 9. As the deformation is very small, angle $AC'C$ may be taken as 45°.

Shearing stress is induced at the faces $DC$ and $AB$

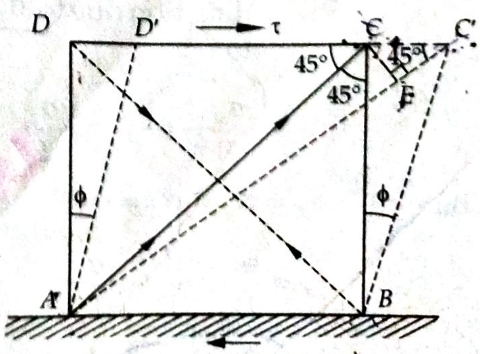

Fig. 9

$$\text{Shear strain } \phi = \frac{CC'}{BC} \qquad \text{...(i)}$$

$$\text{Also shear strain} = \frac{\tau}{C} \qquad \text{...(ii)}$$

In right angled triangle $CEC'$

$$\frac{C'E}{CC'} = \cos 45°$$

or $$CC' = \frac{C'E}{\cos 45°} = \sqrt{2}\,C'E \qquad \left(\because \cos 45° = \frac{1}{\sqrt{2}}\right)$$