$$\eta = 1 - \frac{r_c^{\gamma} - 1}{\gamma r^{\gamma-1}(r_c - 1)}$$

Assumption Made

Following assumptions were made to derive the expression for the air standard efficiency of the diesel cycle.

1. **Air is assumed to be working fluid in the engine cylinder.**
2. **The working fluid (air) is assumed to be a perfect gas and follows the relationship** $pV = mRT$ or $p = \rho RT$.
3. **The compression and the expansion of the working fluid are adiabatic.**
4. **The adiabatic compression and expansion process are frictionless.**
5. **Heat is supplied to the working fluid at constant pressure, by bringing a hot body in contact with the end of the cylinder.**
6. **The heat is rejected at constant volume by bringing the cold body in contact with the cylinder end.**
7. **It is assumed that there are no heat losses from the system to the surrounding.**
8. **The cylinder walls are made of non-conducting material.**
9. **The working fluid has constant specific heats throughout the cycle.**
10. **Take some physical constants.**

    $c_p = 1.005$ kJ/Kg k
    $c_v = 0.717$ kJ/Kg k
    $\gamma = 1.4$
    $M = 29$ Kg/k mol.

9.12 DUAL COMBUSTION CYCLE

Dual combustion cycle is a combination of Otto and Diesel cycle. This is why, it is called mixed cycle. The high speed diesel engines work on this cycle. In high speed diesel engine, when the speed of the engine is more than 350 rpm (say), then it is necessary to start the injection of fuel before the temperature of the compressed air is sufficient to ignite it.

The $p-V$ and $T-s$ diagram of the dual cycle is shown in the Fig. 20 below.

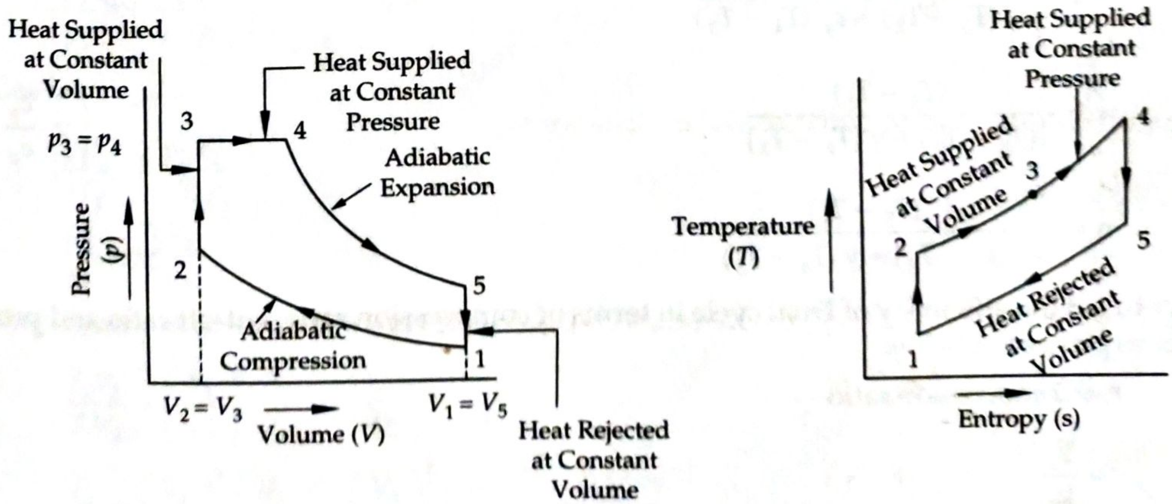

Fig. 20

For the simplicity, air is assumed to be the working fluid. To start with let us assume that the engine cylinder is full of air. The condition of the air at this point 1 is p_1, V_1 and T_1. Let this air be compressed adiabatically to point 2. The air occupies the volume equal to the clearance volume. Let p_2, V_2 and T_2 be the corresponding conditions of the compressed air point 2.

Now, the hot body is brought in contact with the end of cylinder. Heat is supplied to the working fluid, firstly at constant volume process 2 – 3 and then at constant pressure 3 – 4.

At point 4 the hot body is removed from the end of the cylinder. This point is known as the point of cut-off.

Let p_3, V_3, T_3 and p_4, V_4, T_4 be the conditions of air at point 3 and point 4 respectively. When the hot body is removed at point 4, the air is allowed to expand adiabatically to point 5. Let p_5, V_5 and T_5 be the conditions of air at point 5 i.e. at the end of expansion. Now the cold body is brought in contact with the end of the cylinder at point 5. Heat is rejected at constant volume process 5 – 1. This completes the cycles.

Let c_v = Specific heat of air at constant volume

c_p = Specific heat of air at constant pressure.

Let us assume 1 kg of air in the cylinder.

Total Heat supplied = Heat supplied during 2 – 3 + Heat supplied during 3 – 4.

$$c_v (T_3 - T_2) + c_p (T_4 - T_3)$$

Heat rejected = Heat rejected during 5 – 1.

$$= c_v (T_5 - T_1)$$

∴ Work done = Heat supplied – Heat rejected

$$= \left[c_v (T_3 - T_2) + c_p (T_4 - T_3)\right] - c_v (T_5 - T_1)$$

∴ $$\text{Efficiency, } \eta = \frac{\text{Workdone}}{\text{Heat supplied}}$$

$$= \frac{\left[c_v (T_3 - T_2) + c_p (T_4 - T_3)\right] - c_v (T_5 - T_1)}{[c_v (T_3 - T_2) + c_p (T_4 - T_3)]}$$

$$= 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)} \qquad \left[\because \frac{c_p}{c_v} = r\right]$$

or $$\eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)} \qquad \text{...(i)}$$

In order to get the efficiency of Dual cycle in terms of compression ratio, cut-off ratio and pressure ratio, let us proceed as below:

Let r = Compression ratio

$$= \frac{V_1}{V_2}$$

r_c = cut-off ratio

$$= \frac{V_4}{V_3}$$

r_p = Pressure ratio

$$= \frac{p_3}{p_2}$$

We know that,

$$\text{Expansion ratio} = \frac{V_5}{V_4} = \frac{V_1}{V_4} \qquad (\because V_1 = V_5)$$

$$= \frac{V_1}{V_2} \times \frac{V_2}{V_4} \qquad [\text{Multiply and divide by } V_2]$$

$$= \frac{V_1}{V_2} \times \frac{V_3}{V_4} \qquad [\because V_2 = V_3]$$

$$\text{Expansion ratio} = \frac{V_5}{V_4} = \frac{r}{r_c} \qquad \left[\because \frac{V_1}{V_2} = r, \; \frac{V_4}{V_3} = r_c\right]$$

Considering the adiabatic compression process 1 – 2.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

For Ref.

We know that Adiabatic process,

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \qquad [\because pV^{\gamma} = C] \text{ (Adiabatic process)}$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad \text{...(i)}$$

and, Gas law,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\frac{p_2}{p_1} = \frac{T_2V_1}{T_1V_2} \qquad \text{...(ii)}$$

Equating Equations (i) and (ii), we get

$$\frac{T_2V_1}{T_1V_2} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}\left(\frac{V_2}{V_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}\left(\frac{V_1}{V_2}\right)^{-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = r^{\gamma-1} \qquad \left[\because \frac{V_1}{V_2} = r\right]$$

$$T_2 = T_1 r^{\gamma-1} \qquad \text{... (ii)}$$

Considering the constant volume process 2 – 3.

$$\frac{p_2V_2}{T_2} = \frac{p_3V_3}{T_3}$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \qquad (\because V_2 = V_3)$$

$$\frac{T_3}{T_2} = \frac{p_3}{p_2}$$

$$\frac{T_3}{T_2} = r_p \qquad \left[\because \frac{p_3}{p_2} = r_p \text{ pressure ratio}\right]$$

$$T_3 = T_2 r_p \qquad \text{... (iii)}$$

Putting the value of T_2 from Eq. (ii) in Eq. (iii)m we get

$$T_3 = T_1 r^{\gamma-1} r_p \qquad \text{... (vi)}$$

Considering the constant pressure process 3 – 4.

$$\frac{p_3V_3}{T_3} = \frac{p_4V_4}{T_4}$$

For Ref.

We know that Adiabatic process,

$$p_4V_4^{\gamma} = p_5V_5^{\gamma}$$

$$\frac{p_4}{p_5} = \left(\frac{V_5}{V_4}\right)^{\gamma} \qquad \text{...(A)}$$

Gas law,

and,
$$\frac{p_4V_4}{T_4} = \frac{p_5V_4}{T_5}$$

$$\frac{p_4}{p_5} = \frac{T_4V_5}{T_5V_4} \qquad \text{...(B)}$$

Equating *A* and *B*
$$\left(\frac{V_5}{V_4}\right)^{\gamma} = \frac{T_4V_5}{T_5V_4}$$

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma}\left(\frac{V_4}{V_5}\right)$$

$$= \left(\frac{V_5}{V_4}\right)^{\gamma}\left(\frac{V_5}{V_4}\right)^{-1}$$

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1} = \left(\frac{r}{r_c}\right)^{r-1}$$

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \qquad (\because P_3 = P_4)$$

$$\frac{T_4}{T_3} = \frac{V_4}{V_3}$$

$$\frac{T_4}{T_3} = r_c \qquad \left[\because \frac{V_4}{V_3} = r_c \text{ Cut-off ratio}\right]$$

$$T_4 = T_3 r_c \qquad \ldots\text{(v)}$$

Putting the value of T_3 from Eq. (iv) in Eq. (v), we get.

$$T_4 = T_1 r^{\gamma-1} r_p\, r_c \qquad \ldots\text{(vi)}$$

Considering the adiabatic expansion Process 4 – 5.

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1}$$

$$\frac{T_4}{T_5} = \left(\frac{r}{r_c}\right)^{\gamma-1} \qquad \left[\because \frac{V_5}{V_4} = \frac{r}{r_c}\right]$$

$$T_5 = T_4\left(\frac{r_c}{r}\right)^{\gamma-1} \qquad \ldots\text{(vii)}$$

Putting the value of the T_4 from Eq. (vii) in Eq. (vii), we get

$$T_5 = T_1 r^{\gamma-1} r_p\, r_c \left(\frac{r_c}{r}\right)^{r-1}$$

$$T_5 = T_1 r_p\, r_c^{\gamma} \qquad \ldots\text{(viii)}$$

Putting the values of T_2, T_3, T_4 and T_5 from Equations (ii), (iv), (vi) and (viii) in Equation (i), we get

$$= 1 - \frac{T_1 r_p\, r_c^{\gamma} - T_1}{(T_1\, r^{\gamma-1} r_p - T_1\, r^{\gamma-1}) + \gamma\,(T_1\, r^{\gamma-1} r_p\, r_c - T_1\, r^{\gamma-1}\, r_p)}$$

$$= 1 - \frac{T_1 (r_p\, r_c^{\gamma} - 1)}{T_1\, r^{\gamma-1}\left[(r_p - 1) + \gamma\, r_p\, (r_c - 1)\right]}$$

$$\boxed{\eta = 1 - \frac{r_p\, r_c^{\gamma} - 1}{r^{\gamma-1}\left[(r_{p-1}) + \gamma r_p\, (r_c - 1)\right]}}$$

Assumption Made

Following assumptions are made to derive the expression for the air standard efficiency of the dual combustion cycle.

1. Air is assumed to be working fluid in the engine cylinder.
2. The working fluid (air) is assumed to be a perfect gas and follows the relationship

$$pV = mRT \quad \text{or} \quad p = \rho RT$$

3. The compression and the expansion o f the working fluid are adiabatic.
4. The adiabatic compression and expansion process are frictionless.
5. Heat is supplied to the working fluid partly at constant volume, by bring a hot body in contact with the end of the cylinder and remaining at constant pressure.
6. The heat is rejected at constant volume by bringing the cold body in contact with the cylinder end.
7. It is assumed that there are no heat losses from the system to the surrounding.
8. The cylinder walls are made of non-conducting material.
9. The working fluid has constant specific heats throughout the cycle.
10. Take some physical constants.

$$c_p = 1.005 \text{ kJ/Kg k}$$
$$c_v = 0.717 \text{ kJ/Kg k}$$
$$\gamma = 1.4$$
$$M = 29 \text{ Kg/k mol.}$$

9.13 COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

The important variable factors which are used as the basis for comparison of the cycles are:

1. Compression ratio
2. Peak pressure
3. Heat addition
4. Heat rejection, and
5. The Net work

In order to compare the performance of the Otto, Diesel and dual combustion cycles some of the variable factors must be fixed.

Comparison of these three cycles is made for the:

1. Same compression ratio and heat addition.
2. Same compression ratio and Heat rejection.
3. Same peak pressure, peak temperature and heat rejection.
4. Same maximum pressure and heat input.
5. Same maximum pressure and work output.

This analysis will show which cycle is more efficient for a given set of operating conditions.

9.13.1 Same compression ratio and heat addition

For the comparison of Otto, Diesel and Dual combustion cycles, these cycles are drawn on a single $p-V$ and $T-s$ diagram, as shown in Fig. 21(a) and 21(b) respectively for the same compression ratio and heat input. Here,

1−2−3−4−1 represents the otto cycle

1−2−2′−3′−4′−1 represents the dual combustion cycles.

1−2−3″−4″−1 represents the diesel cycle $\frac{V_1}{V_2}$ is the compression ratio for all the three cycles.

From the $T-s$ diagram, it can be seen that Area 5236 = Area 522′3′6′ = Area 523″6″. These area represent heat input which is the same for the three cycles. Area 5146, Area 514′6′ and Area 514″6″ represent heat rejection in Otto cycle, Dual combustion cycle and Diesel cycle respectively.

As the Area 5146 < Area 514′6′ < Area 514″6″.

and, $$\text{Efficiency, } \eta = 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}}$$

therefore, η of the Otto cycle > η of the Dual combustion cycle > η of the Diesel cycle.

$$\boxed{\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}}$$