Similarly, considering the adiabatic expansion process 3 – 4.

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \qquad (\because V_1 = V_4 \text{ and } V_2 = V_3)$$

$$= r^{\gamma-1}$$

$$T_3 = T_4 r^{\gamma-1} \qquad \ldots(v)$$

Putting the values of T_2 and T_3 from equation (ii) and (iii) in equation (i), we get

$$\eta = 1 - \frac{T_4 - T_1}{T_4 r^{\gamma-1} - T_1 r^{\gamma-1}}$$

$$= 1 - \frac{T_4 - T_1}{r^{\gamma-1}[T_4 - T_1]}$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

∴ Air standard efficiency $\boxed{\eta = 1 - \frac{1}{r^{\gamma-1}}}$

Assumption Made

Following assumptions were made to derive the expression for the air standard efficiency of the Otto cycle.

1. Air is assumed to be working fluid in the engine cylinder.
2. The working fluid (air) is assumed to be a perfect gas and follows the relation ship.

   $pV = mRT$

   or $p = \rho RT$
3. The compression and the expansion of the working fluid are adiabatic.
4. The adiabatic compression and expansion processes are frictionless.
5. Heat is supplied to the working fluid at constant volume, by bringing a hot body in contact with the end of the cylinder.
6. The heat is rejected at constant volume by bringing the cold body in contact with the cylinder end.
7. It is assumed that there are no heat losses from the system to the surrounding.
8. The cylinder walls are made of non-conducting material.
9. The working fluid has constant specific heats throughout the cycle.
10. Take some physical constants.

    $c_p = 1.005$ kJ/Kg k

    $c_v = 0.717$ kJ/Kg k

    $\gamma = 1.4$

    $M = 29$ Kg/k mol.

9.11 DIESEL CYCLE

This cycle is very important because the diesel engines are based on this. The first engine working on this cycle was first constructed successfully by Dr. Rudolph Diesel.

The $p-V$ and $T-s$ diagrams of this cycle are shown in Fig. 19.

The cycle consists of two adiabatic processes, one constant pressure process and one constant volume process. For the simplicity, air is assumed to be the working fluid.

In order to calculate the air standard efficiency of cycle, let us consider the engine cylinder to be full of air. The condition of this air is represented by point (1), where the pressure, volume and absolute temperature of the air is p_1, V_1 and T_1 respectively.

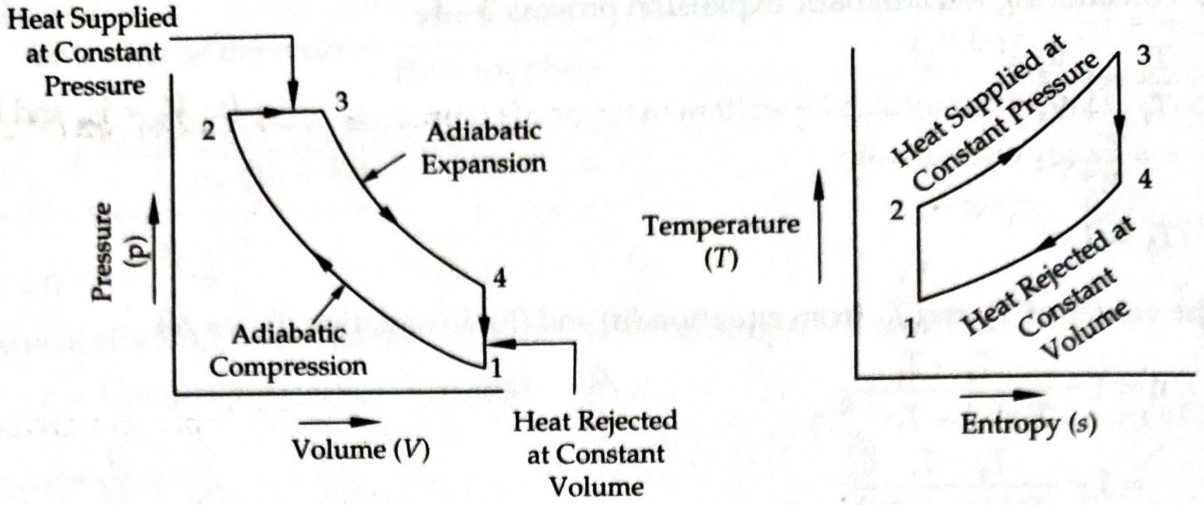

Fig. 19 Diesel cycle

The air at point (1) is compressed adiabatically to point (2). At this point the air occupies the volume equal to the clearance volume. Let p_2, V_2 and T_2 be the pressure, volume and absolute temperature at point (2).

Now, the hot body is brought in contact with the end of the cylinder. The heat is supplied to the air during constant pressure process 2 – 3. At point (3) the hot body is removed from the cylinder end. Let p_3, V_3 and T_3 be the corresponding conditions of air at point (3).

The point (3) at which the supply of heat is stopped is known as the "Point of cut-off".

Further the air is allowed to expand adiabatically to point (4). Let p_4, V_4 and T_4 be the pressure, volume and absolute temperature at that point.

Now the cold body is brought in contact with end of the cylinder.

Heat is rejected at constant volume pressure 4 – 1. The condition of the air becomes the same as that at the beginning of the cycle. The cold body is removed. Thus the cycle is completed.

In this cycle heat is supplied during constant pressure process 2 – 3 and is rejected during constant volume process 4 –1. No heat is transferred during the other two processes i.e. adiabatic processes. This cycle is also known as constant pressure process.

Let c_v = Specific heat of air at constant volume

c_p = Specific heat of air at constant pressure

Considering one kg of air in the engine cylinder.

$\therefore$ Heat supplied $= c_p\,(T_3 - T_2)$

Heat rejected $= c_v\,(T_4 - T_1)$

$\therefore$ Work done = Heat supplied – Heat rejected

$$= c_p\,(T_3 - T_2) - c_v\,(T_4 - T_1)$$

$$\text{Efficiency, } \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_2) - c_v\,(T_4 - T_1)}{c_p\,(T_3 - T_2)}$$

$$= 1 - \frac{c_v\,(T_4 - T_1)}{c_p\,(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{\gamma\,(T_3 - T_2)} \qquad \left[\because \frac{c_p}{c_v} = \gamma\right]$$

or $$\eta = 1 - \frac{T_4 - T_1}{\gamma (T_3 - T_2)} \quad \text{...(i)}$$

Expression of efficiency may also be written in terms of compression ratio and cut-off ratio.

Let r = Compression ratio

r_c = Cut-off ratio.

$\therefore$ $$r = \frac{V_1}{V_2} \text{ and } r_c = \frac{V_3}{V_2}$$

$$\text{Expression ratio} = \frac{V_4}{V_3} = \frac{V_1}{V_3} \quad \left[\text{Multiply and divide by } V_2\right]$$

$$= \frac{V_1}{V_3} \times \frac{V_2}{V_2}$$

or $$= \frac{V_1}{V_2} \times \frac{V_3}{V_2}$$

$$= r \times \frac{1}{r_c} = \frac{r}{r_c} \quad \left[\because \frac{V_1}{V_2} = r, \ \frac{V_3}{V_2} = r_c\right]$$

$$\text{Expansion ratio} = \frac{V_4}{V_3} = \frac{r}{r_c}$$

For Ref.

We know that Adiabatic process,

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad [\because PV^\gamma = C]$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma \quad \text{...(i)}$$

and, Gas Law,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{p_2}{p_1} = \frac{T_2 V_1}{V_2 T_1} \quad \text{...(ii)}$$

Equating equation (i) and (ii), we get

$$\frac{T_2 V_1}{V_2 T_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_1}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_1}{V_2}\right)^{-1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

Considering adiabatic compression process 1 – 2.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 r^{\gamma-1} \qquad \left[\because \frac{V_1}{V_2} = r\right] \qquad \text{...(ii)}$$

Considering the constant pressure process 2 – 3

$$\frac{p_2V_2}{T_2} = \frac{p_3V_3}{T_3}$$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \qquad [\because p_2 = p_3]$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2}$$

$$\frac{T_3}{T_2} = r_c \qquad \left[\because \frac{V_3}{V_2} = r_c\right]$$

$$T_3 = T_2 r_c \qquad \text{...(iii)}$$

Putting the value of T_2 from Eq. (ii) in Eq. (iii).

$$T_3 = T_1 r^{\gamma-1} r_c \qquad \left[\because T_2 = T_1 r^{\gamma-1}\right] \qquad \text{...(iv)}$$

Considering the adiabatic expansion process 3 – 4.

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$= \left(\frac{r}{r_c}\right)^{\gamma-1} \qquad \left[\because \frac{V_4}{V_3} = \left(\frac{r}{r_c}\right)^{\gamma-1}\right]$$

or $$T_3 = T_4\left(\frac{r}{r_c}\right)^{\gamma-1}$$

or $$T_4 = T_3\left(\frac{r_c}{r}\right)^{\gamma-1} \qquad \text{...(v)}$$

Putting the value of T_3 from Equation (iv) in Equation (v)

$$T_4 = T_1 r^{\gamma-1} r_c \left(\frac{r_c}{r}\right)^{\gamma-1}$$

$$T_4 = T_1\, r_c^{\gamma} \qquad \text{...(vi)}$$

Putting the values of T_2, T_3 and T_4 from Equations (ii), (iv) and (vi) in Equation (i), we get

$$\eta = 1 - \frac{T_1 r_c^{\gamma} - T_1}{\gamma\left[T_1 r^{\gamma-1} r_c - T_1 r^{\gamma-1}\right]}$$

$$= 1 - \frac{T_1\left[r_c^{\gamma} - 1\right]}{\gamma\left[T_1 r^{\gamma-1}(r_c - 1)\right]}$$

$$\eta = 1 - \frac{r_c^{\gamma} - 1}{\gamma r^{\gamma - 1}(r_c - 1)}$$

Assumption Made

Following assumptions were made to derive the expression for the air standard efficiency of the diesel cycle.

1. **Air is assumed to be working fluid in the engine cylinder.**
2. **The working fluid (air) is assumed to be a perfect gas and follows the relationship** $pV = mRT$ **or** $p = \rho RT$.
3. **The compression and the expansion of the working fluid are adiabatic.**
4. **The adiabatic compression and expansion process are frictionless.**
5. **Heat is supplied to the working fluid at constant pressure, by bringing a hot body in contact with the end of the cylinder.**
6. **The heat is rejected at constant volume by bringing the cold body in contact with the cylinder end.**
7. **It is assumed that there are no heat losses from the system to the surrounding.**
8. **The cylinder walls are made of non-conducting material.**
9. **The working fluid has constant specific heats throughout the cycle.**
10. **Take some physical constants.**

$c_p = 1.005$ kJ/Kg k

$c_v = 0.717$ kJ/Kg k

$\gamma = 1.4$

$M = 29$ Kg/k mol.

9.12 DUAL COMBUSTION CYCLE

Dual combustion cycle is a combination of Otto and Diesel cycle. This is why, it is called mixed cycle. The high speed diesel engines work on this cycle. In high speed diesel engine, when the speed of the engine is more than 350 rpm (say), then it is necessary to start the injection of fuel before the temperature of the compressed air is sufficient to ignite it.

The $p-V$ and $T-s$ diagram of the dual cycle is shown in the Fig. 20 below.

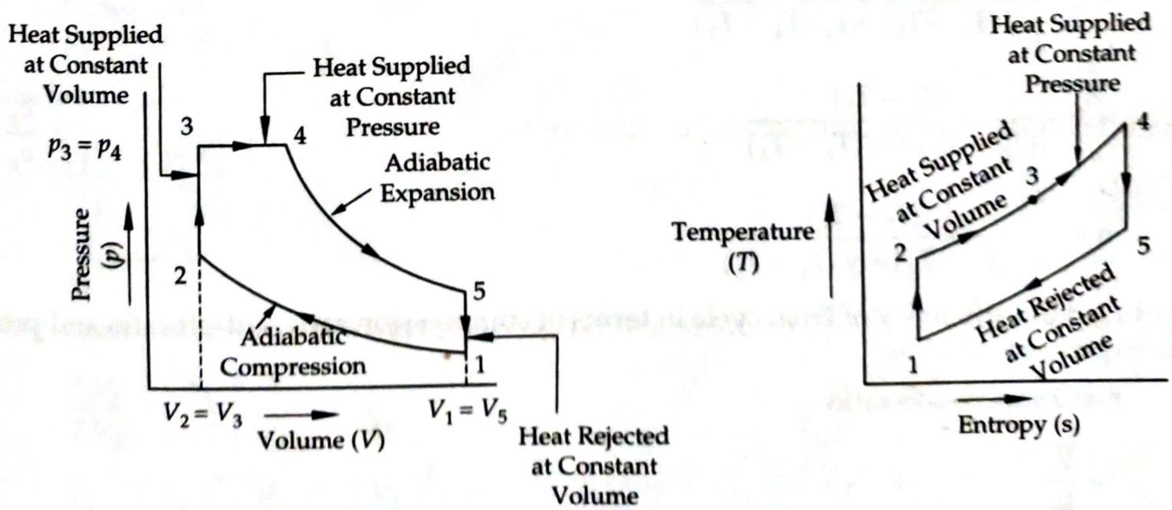

Fig. 20