Method to obtain Prime Implicant

The procedure used for selecting prime implicants is given below:

(1) Each minterm/maxterm should be expressed by its binary representation.

(2) The minterms/maxterms should be arranged according to increasing index (index means the number of 1's in a minterm/maxterm). Separate each set of minterms/maxterms possessing the same index by lines.

(3) Each index i.e., 'n' should be compared with each index (n + 1). For each pair of terms that can combine, the newly formed terms should be stated. If two minterms/maxterms differ in only one variable, that variable should be removed and a dash (–) placed at that position. It shows the pairing of '2' cell combination.

After all the pairs of terms with indices n & (n + 1) have been considered, a line should be drawn under the last term.

(4) When the above process has been repeated for all the groups of terms, one stage of elimination will have been completed.

(5) The next step of elimination or matching process should be repeated for the new terms. According to this step, two terms can be combined only when they have dashes in the same position. It results in pairing of '4' cells combination.

(6) The process is repeated until no new list can be found.

(7) All terms which remain unchecked (do not match) during the process are considered as prime implicants.

Method to obtain Essential Prime Implicant

(1) Here, there is a formation of prime implicant chart. For this, prime implicants shouldbe represented in rows and each minterm/maxterm of the function in a column.

(2) Crosses should be placed in each row to show the composition of minterms that makes the prime implicants.

(3) A completely prime implicants table should be checked for column containing only a single cross. Prime implicants that cover minterm/ maxterms witha single cross in their column are called essential Prime implicants.

e.g., Solve the given function using QM method

$$y = \Sigma m(0, 1, 5, 7)$$

Firstly these minterms are represented in binary representation (as maximum term is 7, so '3' variables are used *i.e., A, B, C*)

Minterms Binary representation

Minterms	Binary representation		
	A	B	C
0	0	0	0
1	0	0	1
5	1	0	1
7	1	1	1

Next step is to dete rmine the prime implicant. For this each bi representation should be written according to number of ones in incre order.

Index	Minterm	Variables		
		A	B	C
0	0	0	0	0
1	1	0	0	1
2	5	1	0	1
3	7	1	1	1

Next step is to check the pairing of ones for this each index 'n' is comp to $(n + 1)$ index and a dash is placed, if the two minterms differ in only one vari

Index	minterm	Variables		
		A	B	C
0	(0, 1)	0	0	—
1	(1, 5)	—	0	1
2	(5, 7)	1	—	1

Next step is to check for next one's combination i.e., 4 ones. Here, no dashes are same, so no further pairing is formed.

So, (0, 1) (1, 5), (5, 7) minterms forms the prime implicants. The minte corresponding to Prime implicants are $\bar{A}\,\bar{B}$, $\bar{B}\,C$ & $A\,C$.

The next step is to form the prime implication chart which is shown as

Prime Implicants	Minterms			
	0	1	5	7
(0, 1)	⊗	X		
(1, 5)		×	×	
(5, 7)			×	⊗

The encircled cross indicate the single cross in that column. There encirc cross is known as essential Prime implicant. So it is given as $\bar{A}\,\bar{B}$ & AC.

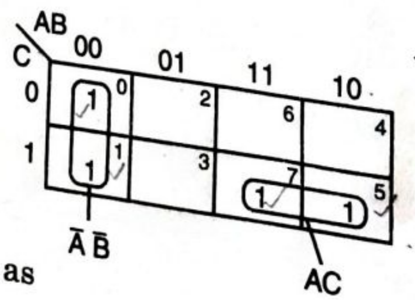
So the final output includes the essential Prime implicant and all minterm is covered in essential prime implicant.

i.e.,

$$y = \bar{A}\,\bar{B} + A\,C$$

It can also be verified using K-map For this we require '3' variable K-m

i.e.,



So the output is given as

$$y = \bar{A}\bar{B} + AC$$

which is same as solved by QM method

EXAMPLE. Solve by Quine-Mcclusky Minimization Technique

$$Y(A, B, C, D) = \Sigma m(0, 1, 3, 7, 8, 9, 11, 15)$$

Solution: Step 1

Put binary representation of all the miniterms and arrange them according to index wise. Index indicates no. of 1's present in a minterm.

$Y(A, B, C, D) = \Sigma m$ (0000, 0001, 0011, 0111, 1000, 1001, 1011, 1111)

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

Index 0 Index1 Index2 Index3 Index1 Index2 Index3 Index4

Index (Group)	Minterm	Variables			
		A	B	C	D
0	0	0	0	0	0 ✓
1	1	0	0	0	1 ✓
~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~ ✓
	8	1	0	0	0 ✓
2	3	0	0	1	1 ✓
	9	1	0	0	1 ✓
3	7	0	1	1	1 ✓
	11	1	0	1	1 ✓
4	15	1	1	1	1 ✓

Step 2: Compare each minterm in a group n with each minterm in group $n + 1$ and identify (Match the pair of minterms in which only 1 variable changes).

If a matched pair is founded, Then a ✓ Mark is applied to both individual minterms available in pair.

Comparison and identification of matched pairs (for making group of two is)

Index (Group)	Minterms (Grouping of Two 1's	Variables			
		A	B	C	D
0	0, 1	0	0	0	—✓
	0, 8	—	0	0	0 ✓
1	1, 3	0	0	—	1✓
	1, 9	—	0	0	1✓
	8, 9	1	0	0	—✓
2	3, 7	0	—	1	1✓
	3, 11	—	0	1	1✓
	9, 11	1	0	—	1✓
3	7, 15	—	1	1	1✓
	11, 15	1	—	1	1✓

Step 3: Comparison and identification of matched pairs (for making gro of four 1's)

Index (Group)	Minterms (Grouping of four is	Variables			
		A	*B*	*C*	*D*
0	(0, 1, 8, 9)	—	0	0	—
	(0, 8, 1, 9)	—	0	0	—
1	(1, 3, 9, 11)	—	0	—	1
	(1, 9, 3, 11)	—	0	—	1
2	(3, 7, 11, 15)	—	—	1	1
	(3, 11, 7, 15)	—	—	1	1

Step 4: Comparison and identification of matched pair (for making group eight 1's)

⇓

As no matching possible

⇓

Hence, identification process is complete

Comparison and identification of match pair process proceeds until no furthe combination of minterm groups is possible.

Step 5: Now all minterms groups (single, grouping of two 1's and groupin of four 1's) are checked for tickmark. Any minterm which is not tick marked i called as prime implicant.

e.g. Minterms (Single) : All are tick marked

(In a pair of two 1's) : All are tick marked

(In a pair of four 1's): No one is tick marked.

Hence (0, 1, 8, 9), (1, 3, 9, 11), (3, 7, 11, 15) groups of minterms are makin prime implicants $\bar{B}\,\bar{C}$, $\bar{B}$ D, C D respectively.

$$Y = \bar{B}\,\bar{C} + \bar{B}\,D + C\,D$$

Step 6 : Preparation of prime implicant table

PI terms	Decimal numbers (Minterm groups)	All minterms (In a given function)							
		0	1	3	7	8	9	11	15
$\bar{B}\,\bar{C}$	(0, 1, 8, 9)	⊗	×			⊗	×		
$\bar{B}\,D$	(1, 3, 9, 11)		×	×			×	×	
CD	(3, 7, 11, 15)			×	⊗	●		×	⊗
		✓			✓	✓			✓

Minterms corresponding to single × are encircled and called as essential PI and tick marked is but below columns.

Searching of essential prime implicant

Minterms 0, 8 are contained in only one prime implicant *i.e.*, $\bar{B}\,\bar{C}$ and minterms 7, 15 are contained in only one PI *i.e.*, *CD*.

Hence these two PI are called as essential prime implicants (E PI).

In this way—

Minimized expression is–

$\boxed{Y = \bar{B}\,\bar{C} + CD}$ These two ES PI covers all minterms 0, 1, 3, 7, 8, 9, 11, 15

EXAMPLE 2.16. Solve the following function by K-map.

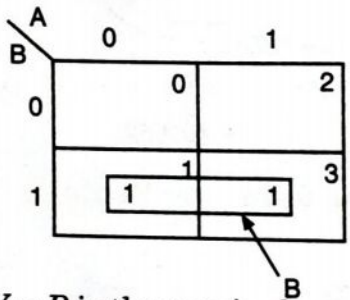
(i) $Y = \Sigma m(1, 3)$

(ii) $Y = \Sigma m(2, 3, 5)$

(iii) $Y = \Sigma m(4, 12, 13, 15)$

Solution : (*i*) $Y = \Sigma m(1, 3)$

Denotes the minterms expression and 1, 3 is included in '2' variable because '1' notation is 0 1 and '3' is '1 1' so two binary bits are used, so two variable K-map is used.



So $Y = B$ is the required answer.

$= B\,.\,(1) = B$

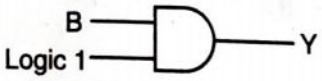
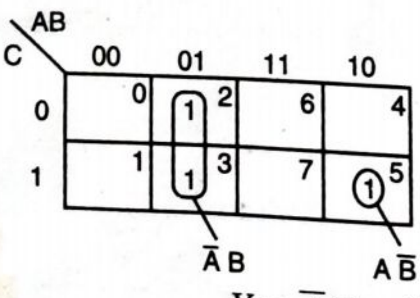


Fig. 2.50.

(*ii*) $Y = \Sigma m(2, 3, 5)$

Maximum minterms is '5' so represented by '1 0 1' so 3-variable 'K' map is used.



$Y = \bar{A}B + A\bar{B}C$

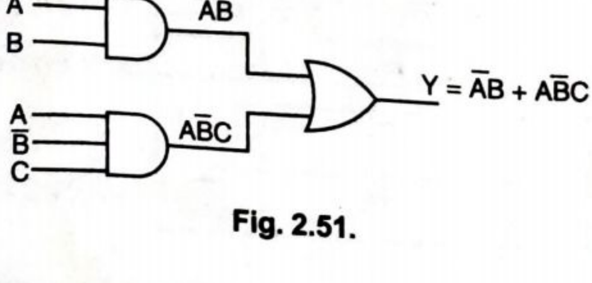
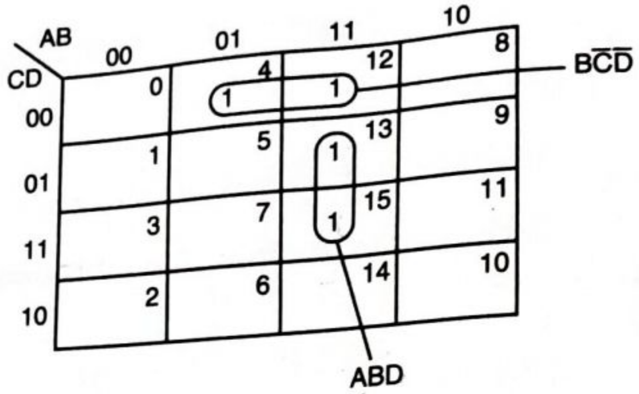


Fig. 2.51.

(*iii*) $Y = \Sigma m(4, 12, 13, 15)$

Minterm 12, 13, 15 can be represented by 4 bits so 4-variable K-map required



$$Y = B\overline{C}\,\overline{D} + ABD$$

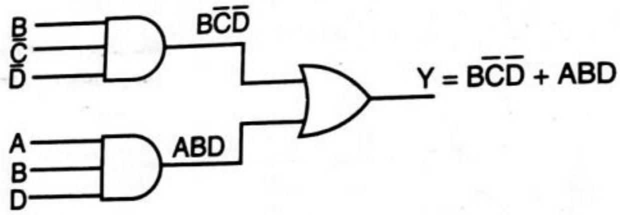


Fig. 2.52.

EXAMPLE 2.17. Solve the following Boolean function by K-map.

(*i*) $Y = \pi M(0, 1)$

(*ii*) $Y = \pi M(4, 5, 10, 12)$

(*iii*) $Y = \pi M(0, 2, 3, 7) + \phi(4)$

(*iv*) $Y = \pi M(1, 2, 3, 5) + d(0, 6)$

Solution: (*i*) $Y = \pi M(0, 1)$

It is the maxterm expression and fill it by '0' value, i.e.

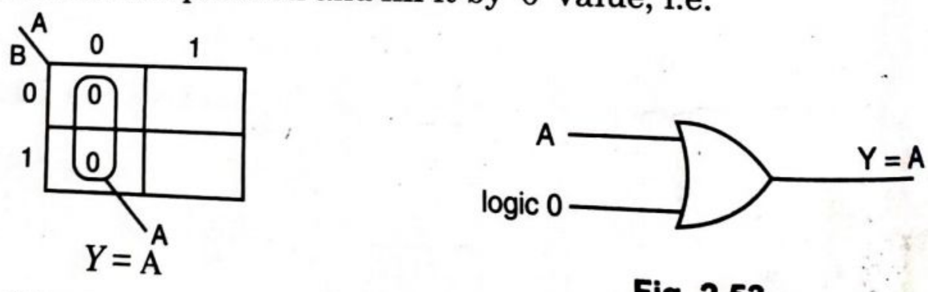
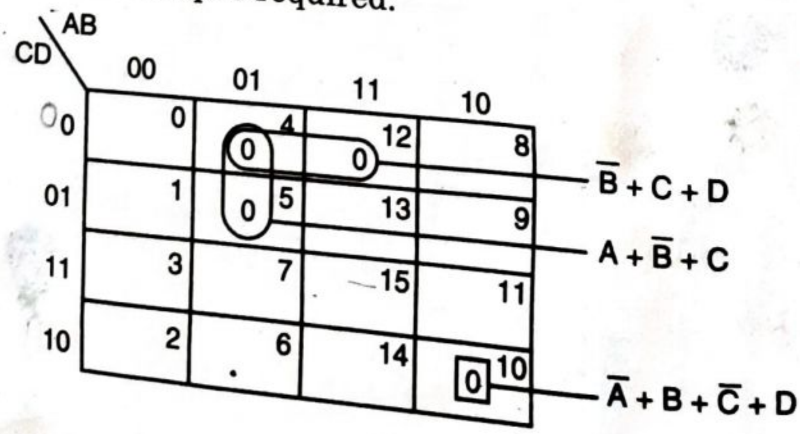


Fig. 2.53.

(*ii*) $Y = \pi M(4, 5, 10, 12)$

Here '4' variable K-map is required:



$$Y = (\overline{B} + C + D)\,(A + \overline{B} + C)\,(\overline{A} + B + \overline{C} + D)$$

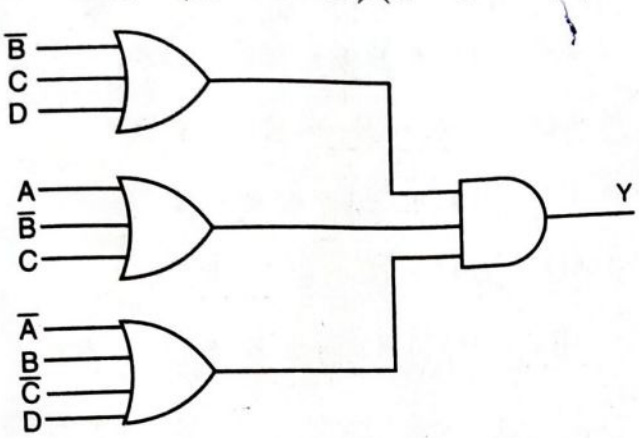


Fig. 2.54.

(*iii*) $Y = \pi M(0, 2, 3, 7) + \phi(4)$

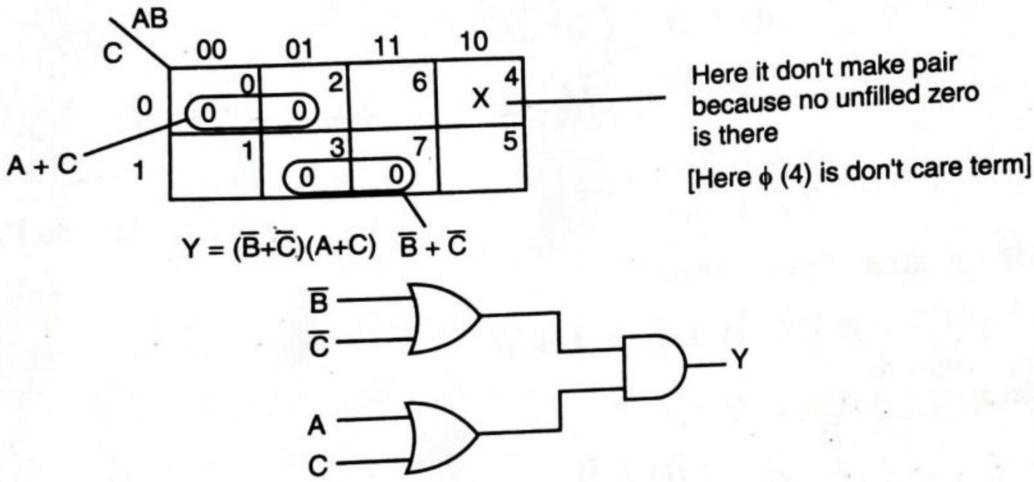


Fig. 2.55.

(*iv*) $Y = \pi M(1, 2, 3, 5) + d(0, 6)$

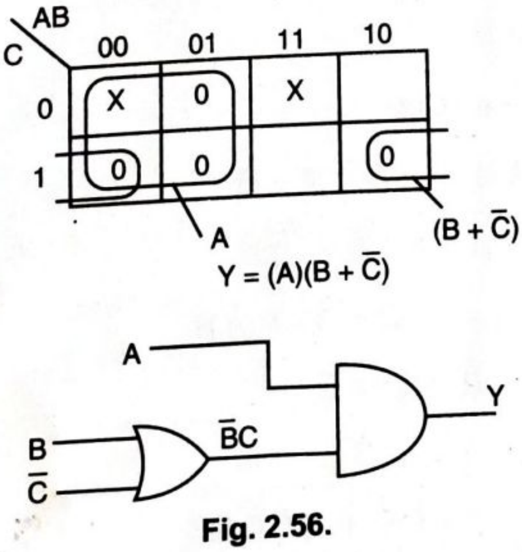


Fig. 2.56.

EXAMPLE 2.18. Prove that

$$(A + \overline{B} + \overline{C})\,(A + \overline{B}C) = A + \overline{B}C$$

Solution: $(A + \overline{B} + \overline{C})\,(A + \overline{B}C)$

$$= AA + A\overline{B}C + \overline{B}A + \overline{B}\,\overline{B}C + \overline{C}A + \overline{C}BC$$

$$= A + A\overline{B}C + \overline{B}A + \overline{B}C + \overline{C}A \quad [\because A.A = A, \overline{C}\,\overline{B}($$

$$= A[1 + \overline{B}C + \overline{B}] + \overline{B}C + \overline{C}A$$

$$= A[1 + \overline{B}C + \overline{B}] + \overline{C}A + \overline{B}C$$

$$= A[1 + \overline{B}C + \overline{B} + \overline{C}] + \overline{B}C$$

$$= A[(1 + \overline{B})(1 + \overline{C}) + \overline{B} + \overline{C}] + \overline{B}C$$

$$[\because (1 + \overline{B}C) = (1 + \overline{B})(1$$

$$= A[(1)(1) + \overline{B} + \overline{C}] + \overline{B}C \quad [\because 1 + \overline{B} = 1 \text{ and } 1 + ($$

$$= A[1 + \overline{B} + \overline{C}] + \overline{B}C$$

$$= A[1 + \overline{C}] + \overline{B}C \qquad [\because 1 + \overline{C}$$

$$= A + \overline{B}C$$

= R. H. S

Hence Pro

EXAMPLE 2.19. Prove that

$$\overline{A}B + \overline{A}\,\overline{B} + \overline{B} = \overline{A} + \overline{B}$$

Solution: $\overline{A}B + \overline{A}\,\overline{B} + \overline{B}$

$$= \overline{A}(B + \overline{B}) + \overline{B}$$

$$= \overline{A}(1) + \overline{B}$$

$$= \overline{A} + \overline{B} \qquad [\because B + \overline{B}$$

= R.H.S

EXAMPLE 2.20. Prove that

$$AB + A\overline{B} + \overline{A}B = A + B$$

Solution:

$$AB + A\overline{B} + \overline{A}B$$

$$= A(B + \overline{B}) + \overline{A}B$$

$$= A(1) + \overline{A}B$$

$$= A + \overline{A}B \qquad [\because B + \overline{B}$$

$$= (A + \overline{A})(A + B)$$

$$= (1) \cdot (A + B)$$

$$= A + B$$

= R.H.S $\qquad [\because A + \overline{A}$

EXAMPLE 2.21. Simplify the following expressions :

(i) $B(A+C)+A\overline{B}+B\overline{C}+C$

(ii) $ABC+\overline{A}BC+A\overline{B}C+\overline{A}\,\overline{B}C+AB\overline{C}$

(iii) $\overline{A}D+ABD$

(iv) $\overline{(\overline{A}+C).(B+\overline{D})}$

Solution: (i) $B(A+C)+A\overline{B}+B\overline{C}+C$

$$= BA+BC+A\overline{B}+B\overline{C}+C$$
$$= BA+BC+A\overline{B}+B\overline{C}+C$$
$$= A(B+\overline{B})+B(C+\overline{C})+C$$
$$= A(1)+B(1)+C \qquad [\because B+\overline{B}=1, C+\overline{C}=1]$$
$$= A+B+C$$

(ii) $ABC+\overline{A}BC+A\overline{B}C+\overline{A}\,\overline{B}C+AB\overline{C}$

$$= BC(A+\overline{A})+\overline{B}C(A+\overline{A})+AB\overline{C}$$
$$= BC(1)+\overline{B}C(1)+AB\overline{C} \qquad [\because A+\overline{A}=1]$$
$$= BC+\overline{B}C+AB\overline{C}$$
$$= C(B+\overline{B})+AB\overline{C} \qquad [\because C+\overline{C}=1]$$
$$= C+AB\overline{C}$$
$$= (C+\overline{C})(C+AB)$$
$$= (1)(C+AB)$$
$$= C+AB$$

(iii) $\overline{A}D+ABD$

$$= D(\overline{A}+AB) \qquad [\because \overline{A}+AB=(\overline{A}+A)(\overline{A}+B)]$$
$$= D[(\overline{A}+A)(\overline{A}+B)]$$
$$= D[(1)(\overline{A}+B)]$$
$$= D[\overline{A}+B]$$
$$= \overline{A}D+BD$$

(iv) $\overline{(\overline{A}+C).(B+\overline{D})}$

$$= \overline{(\overline{A}+C)}+\overline{(B+\overline{D})} \qquad [\because \overline{A.B}=\overline{A}+\overline{B}]$$
$$= \overline{\overline{A}}.\overline{C}+\overline{B}.\overline{\overline{D}} \qquad [\because \overline{A+B}=\overline{A}.\overline{B}]$$
$$= A.\overline{C}+\overline{B}.D \qquad [\because \overline{\overline{A}}=A]$$
$$= A\overline{C}+\overline{B}D$$

EXAMPLE 2.22. Solve the following functions :

(i) $Y = (B + CA)(C + \bar{A}B)$

Solution:

$$Y = BC + B\bar{A}B + CAC + CA\bar{A}B$$

$$= BC + B\bar{A} + CA + 0 \qquad [\because A\bar{A} = 0, B.B = B, C.C = \ldots$$

$$= BC + B\bar{A} + CA$$

(ii) $Y = AB + AC + BD + CD$

Ans.

$$Y = AB + AC + BD + CD$$

$$= A(B + C) + D(B + C)$$

$$= (A + D)(B + C)$$

(iii) $Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$

Ans.

$$Y = \bar{A}\bar{B}\bar{C}(\bar{D} + D) + A\bar{B}\bar{C}(\bar{D} + D)$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} \qquad [\because D + \bar{D} = 1$$

$$= \bar{B}\bar{C}[\bar{A} + A] \quad [\because A + \bar{A} = 1]$$

$$= \bar{B}\bar{C}$$

(iv) $Y = \overline{AB} + \overline{\bar{A} + B}$

Ans.

$$Y = \bar{A} + \bar{B} + (\bar{\bar{A}}.\bar{B}) \qquad [\because \overline{A + B} = \bar{A}.\bar{B}$$

$$\overline{A.B} = \bar{A} + \bar{B}]$$

$$= \bar{A} + \bar{B} + A\bar{B}$$

$$= \bar{A} + \bar{B}[1 + A]$$

$$= \bar{A} + \bar{B} \; [\because 1 + A = 1]$$

$$= \overline{AB} \quad [\because \bar{A} + \bar{B} = \overline{AB}]$$

(v) $Y = \overline{AB.(\bar{B}C + AC)}$

Ans.

$$Y = \overline{AB.(\bar{B}C + AC)}$$

$$= \overline{AB} + \overline{(\bar{B}C + AC)} \qquad [\because \overline{A.B} = \bar{A} + \bar{B}]$$

$$= \bar{A} + \bar{B} + \left(\overline{\bar{B}C}.\overline{AC}\right)$$

$$= (\bar{A} + \bar{B}) + (B + \bar{C})(\bar{A} + \bar{C})$$

EXAMPLE 2.23. Solve the following problems using K-maps

(i) $f(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}B + AB\bar{C} + AC$

(ii) $f(A, B, C) = \bar{A}B + B\bar{C} + BC + A\bar{B}\bar{C}$

(iii) $f = \Sigma m(0, 2, 3, 6, 7, 8, 9, 10, 13)$

(iv) $f(P, Q, R, S) = \Sigma m(0, 1, 2, 4, 6, 8, 10, 12, 14)$

Solution : (i) $f(A, B, C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B + AB\overline{C} + AC$

Convert these terms into standard SOP form to find out the minterms:

Binary value   Minterm

$$\overline{A}\,\overline{B}\,\overline{C} = 0\,0\,0 \qquad (0)_{10}$$

$$\overline{A}B = \overline{A}B(C + \overline{C}) = \overline{A}BC + \overline{A}B\overline{C}$$

$$\overline{A}BC \downarrow 011 \downarrow (3)_{10} \qquad \overline{A}B\overline{C} \downarrow 010 \downarrow (2)_{10}$$

$$AB\overline{C} = 110 = (6)_{10}$$

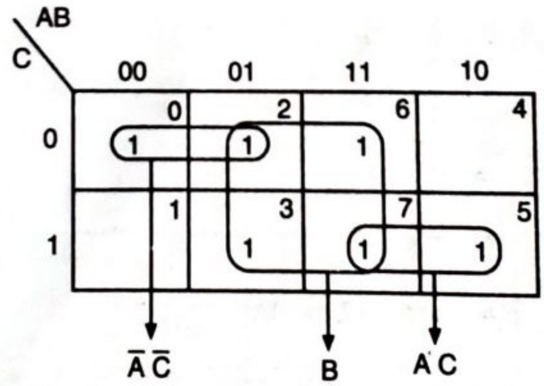
$$AC = A(B + \overline{B})C = ABC + A\overline{B}C$$

$$ABC \downarrow 111 \downarrow (7)_{10} \qquad A\overline{B}C \downarrow 101 \downarrow (5)_{10}$$

So, the minterms are given as:

$\Sigma m(0, 2, 3, 5, 6, 7)$

This can be solve by 3-variable K-map because the maximum decimal term is 7.



So, the final result is given as:

$$f(A, B, C) = B + AC + \overline{A}\,\overline{C}$$

(ii) $f(A, B, C) = \overline{A}B + B\overline{C} + BC + A\overline{B}\,\overline{C}$

Ans. Convert the following terms into standard SOP form

$$\overline{A}B \Rightarrow \overline{A}B(C + \overline{C}) \qquad \overline{A}BC + \overline{A}B\overline{C}$$

$$\overline{A}BC \downarrow 011 \downarrow \qquad \overline{A}B\overline{C} \downarrow 010 \downarrow$$

Minterm → 3   2

$$B\overline{C} \rightarrow (A+\overline{A})\,B\overline{C} = \begin{array}{ccc} AB\overline{C} & + & \overline{A}B\overline{C} \\ \downarrow & & \downarrow \\ 110 & & 010 \\ \downarrow & & \downarrow \\ 6 & & 2 \end{array}$$

Minterm →

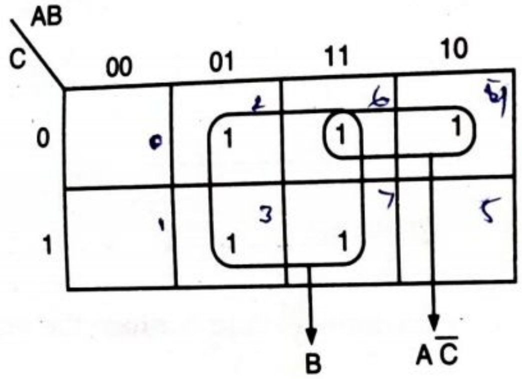
$$BC \rightarrow (A+\overline{A})\,BC = \begin{array}{ccc} ABC & + & \overline{A}BC \\ \downarrow & & \downarrow \\ 111 & & 011 \\ \downarrow & & \downarrow \\ 7 & & 3 \end{array}$$

Minterm →

$$\overline{A}\,\overline{B}\,\overline{C} \rightarrow 100 \rightarrow 4 \text{ (Minterm)}$$

Minterms are given as $\Sigma m\,(2, 3, 4, 6, 7)$ [Repeat term, is taken once]

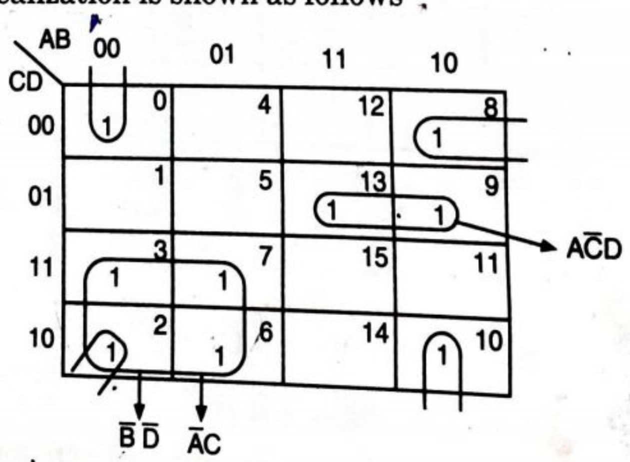
The K-map realization is shown as follows :



$$f(A, B, C) = B + A\overline{C}$$

(*iii*) $f = \Sigma m\,(0, 2, 3, 6, 7, 8, 9, 10, 13)$

The K-map realization is shown as follows

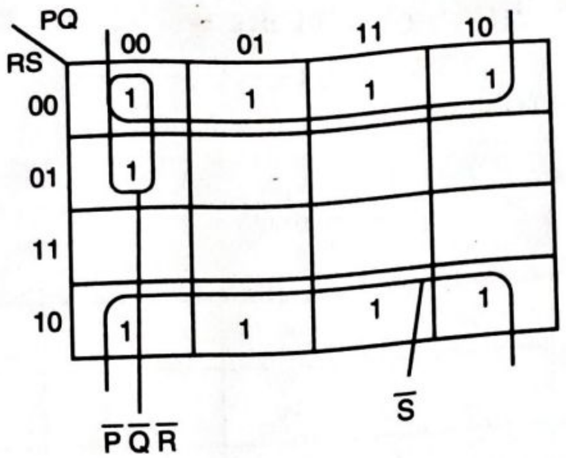


The output is given as :

$$f(A, B, C, D) = \overline{A}C + A\overline{C}D + \overline{B}\,\overline{D}$$

(*iv*) $f(P, Q, R, S) = \Sigma m\,(0, 1, 2, 4, 6, 8, 10, 12, 14)$

The K-map is shown as follows



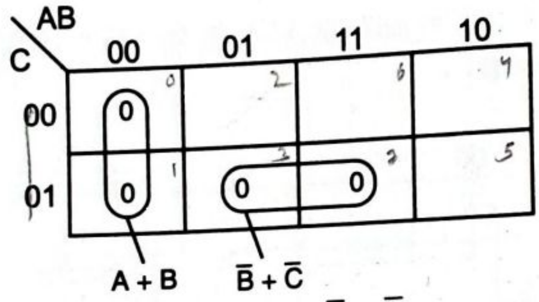
The output is given as :

$$f(P, Q, R, S) = \overline{S} + \overline{P}\ \overline{Q}\ \overline{R}$$

EXAMPLE 2.24 Simplified the function using QM method & k-map.

$$Y = \pi M\ (0, 1, 3, 7).$$

Solution : Using k-map



$$Y = (A + B)\,(\overline{B} + \overline{C})$$

Using QM-Method

(*i*) Convert the max terms into their binary equivalent and arrange them according to increasing order of their index

Index	Maxterm	Variable			
		A	B	C	
0	0	0	0	0	√
1	1	0	0	1	√
2	3	0	1	1	√
3	7	1	1	1	√

(*ii*) Compare each maxterm in lower index with each term in higher succeeding index. If there is change in only single bit, mark dash (—) in that position in next table and mark right (√) on both terms in the above table.

Index	Maxterm	Variables		
		A	B	C
0	(0, 1)	0	0	—
1	(1, 3)	0	—	1
2	(3, 7)	—	1	1

Hence no more further grouping is possible

$$Y = (A + B)\,(A + \overline{C})\,(\overline{B} + \overline{C})$$

(*iii*) Find essential prime implicants

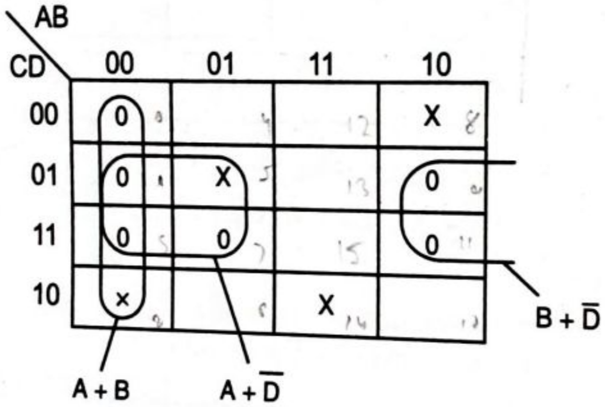
Prime implicants \ Maxterm	0	1	3	7
$A + B$	⊗	×		
$A + \overline{C}$		×	×	
$\overline{B} + \overline{C}$			×	⊗

Hence $\boxed{Y = (A + B)\,(\overline{B} + \overline{C})}$

EXAMPLE 2.25. Simplified the given function using k-map & Q-M method

$$Y = \pi M\,(0, 1, 3, 7, 9, 11) + d(2, 5, 8, 14)$$

Solution : Using k-map



$$Y = (A + B)\,(A + \overline{D})\,(B + \overline{D})$$

Using QM Method

(*i*) Convert the maxterms and don't care conditions into their binary equivalent and arrange them according to their index.

Index	Maxterm	A	B	C	D	
0	0	0	0	0	0	√
1	1	0	0	0	1	√
	2*	0	0	1	0	√
	8*	1	0	0	0	√
2	3	0	0	1	1	√
	5*	0	1	0	1	√
	9	1	0	0	1	√
3	7	0	1	1	1	√
	11	1	0	1	1	√
Prime implicant	14*	1	1	1	0	$\overline{A} + \overline{B} + \overline{C}$ ...

(*ii*) Compare each term in lower index with each term in higher succeeding index including don't care conditions also. If there is change in only one bit, mark dash (—) in that position in next table and mark right (√) on both terms in the above table.

Index	Group	Variables				
		A	B	C	D	
0	(0, 1)	0	0	0	—	√
	(0, 2*)	0	0	—	0	√
	(0, 8*)	—	0	0	0	√
1	(1, 3)	0	0	—	1	√
	(1, 5*)	0	—	0	1	√
	(1, 9)	—	0	0	1	√
	(2*, 3)	0	0	1	—	√
	(8*, 9)	1	0	0	—	√
2	(3, 7)	0	—	1	1	√
	(3, 11)	—	0	1	1	√
	(5*, 7)	0	1	—	1	√
	(9, 11)	1	0	—	1	√

(*iii*) Repeat the step (*ii*) until no more grouping is possible.

Index	Group	Variables			
		A	B	C	D
0	(0, 1, 2*, 3)	0	0	—	—
	(0, 1, 8*, 9)	—	0	0	—
	(0, 2*, 1, 3)	0	0	—	—
	(0, 8*, 1, 9)	—	0	0	—
1	(1, 3, 5*, 7)	0	—	—	1
	(1, 3, 9, 11)	—	0	—	1
	(1, 5*, 3, 7)	0	—	—	1
	(1, 9, 3, 11)	—	0	—	1

(*iv*) Hence no more grouping is possible we write equation covering all maxterms and known as prime implicant equation. The procedure is similar to SOP form the only change is that the variable having value '1' is written in compliment form and variable having value '0' is written in uncomplement form.

$$Y = (A + B)\,(B + C)\,(A + \overline{D})\,(B + \overline{D})$$

The term left in first table i.e., $(\overline{A} + \overline{B} + \overline{C} + D)$ is not included because it is don't care conditions.

$$Y = (A + B)\,(B + C)\,(A + \overline{D})\,(B + \overline{D})$$

(*v*) Now we find the essential prime implicant from the prime implicant chart: Check the maxterm which have only single × and circle it and these maxterms are essential prime implicants and should be come in final reduced equation.

$$Y = (A+B)(B+C)(A+\overline{D})(B+\overline{D})$$

Maxterm / Prime implicants	0	1	2*	3	5*	7	8*	9	11	[illegible]
$A+B$	×	×	⊗	×			⊗	×		
$B+C$	×	×								
$A+\overline{D}$		×		×	⊗	⊗		×	⊗	
$B+\overline{D}$		×		×						
$A+\overline{B}+\overline{C}+D$										⊗

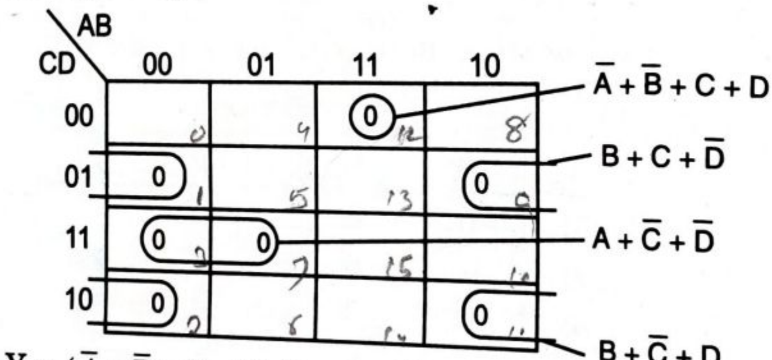
Check the maxterm which have only single × and circle it and the maxterms are essential prime implicants and should be come in final reduc equation.

$$Y = [(A+B)/(B+C)](A+\overline{D})(B+\overline{D})$$

EXAMPLE 2.26 Solve the given function using k-map & QM method

$$Y = \pi M(1, 2, 3, 7, 9, 10, 12)$$

Solution : Using k-map



$$Y = (\overline{A}+\overline{B}+C+D)(B+C+\overline{D})(A+\overline{C}+\overline{D})(B+\overline{C}+D)$$

Using QM method

(*i*) Convert the maxterm into their binary equivalent and arrange the according to increasing order of their index.

Index	Group	A	B	C	D	
1	1	0	0	0	1	√
	2	0	0	1	0	√
2	3	0	0	1	1	√
	9	1	0	0	1	√
	10	1	0	1	0	√
Prime implicant —	12	1	1	0	0	$\overline{A}+\overline{B}+C+$
3	7	0	1	1	1	√

(*ii*) Compare each maxterm of lower index with each maxterm of high succeeding index and if there is an change in single bit only then ma dash (—) on that position in next table and mark right (√) on bo maxterms in above table.

Index	Group	Variable			
		A	B	C	D
1	(1, 3)	0	0	—	1
	(1, 9)	—	0	0	1
	(2, 3)	0	0	1	—
	(2, 10)	—	0	1	0
2	(3, 7)	0	—	1	1

No more grouping is possible and the equation is given by

$$Y = (\bar{A}+\bar{B}+C+D)\ (A+B+\bar{D})\ (B+C+\bar{D})$$
$$(A+B+\bar{C})\ (B+\bar{C}+D)\ (A+\bar{C}+\bar{D})$$

[$\bar{A}+\bar{B}+C+D$ is included because 12 Maxterm term is not present in any grouping]

(*iii*) Find out essential prime implicants from the prime implicant chart.

Maxterm → / Prime implicants ↓	1	2	3	7	9	10	12
$(\bar{A}+\bar{B}+C+D)$							⊗
$(A+B+\bar{D})$	×		×				
$(B+C+\bar{D})$	×				⊗		
$(A+B+\bar{C})$		×	×				
$(B+\bar{C}+D)$		×				⊗	
$(A+\bar{C}+\bar{D})$			×	⊗			

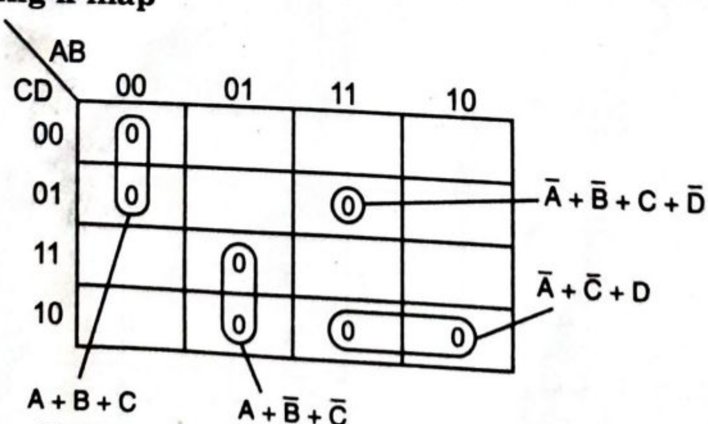
The prime implicant which are encircle are come in final equation. Hence

$$Y = (\bar{A}+\bar{B}+C+D)\ (B+C+\bar{D})\ (B+\bar{C}+D)\ (A+\bar{C}+\bar{D})$$

EXAMPLE 2.27 Solve the given function

$$Y = \pi M\ (0, 1, 6, 7, 10, 13, 14)$$

Solution : Using k-map



$$Y = (\bar{A}+\bar{B}+C+\bar{D})\ (\bar{A}+\bar{C}+D)\ (A+\bar{B}+\bar{C})\ (A+B+C)$$

Using Q-M method

(*i*) Convert the maxterm into their binary equivalent and arrange th according to increasing order of their index.

Index	Group	A	B	C	D	
		Variable				
0	0	0	0	0	0	√
1	1	0	0	0	1	√
2	6	0	1	1	0	√
	10	1	0	1	0	√
3	7	0	1	1	1	√
Prime implicant →	13	1	1	0	1	$\overline{A}+\overline{B}+C+$
	14	1	1	1	0	√

(*ii*) Now compare each maxterm of lower index with each maxterm of high succeeding index and if there is an change in single bit only, then ma dash (—) on that position in next table and mark right (√) on bo maxterms in above table.

Index	Group	Variables			
		A	B	C	D
0	(0, 1)	0	0	0	—
2	(6, 7)	0	1	1	—
	(6, 14)	—	1	1	0
	(10, 14)	1	—	1	0

No more grouping is possible.

$$Y = (\overline{A} + \overline{B} + C + \overline{D})\,(A + B + C)\,(A + \overline{B} + \overline{C})$$

$$(\overline{B} + \overline{C} + D)\,(\overline{A} + \overline{C} + \ldots$$

[$\overline{A} + \overline{B} + C + D$ is included because Maxterm 13 is no involved with an pairing]

(*iii*) Now we find the essential prime implicant from the prime implicant char table.

Maxterm → / Prime implicants ↓	0	1	6	7	10	13	14
$\overline{A} + \overline{B} + C + \overline{D}$						⊗	
$A + B + C$	⊗	⊗					
$A + \overline{B} + \overline{C}$			×	⊗			
$\overline{B} + \overline{C} + D$			×				×
$\overline{A} + \overline{C} + D$					⊗		×

Encircle the cross (×) which is single in an each column and then the prime implicants corresponding to that row is known as essential prime implicants, hence final result is

$$Y = (\overline{A} + \overline{B} + C + \overline{D})(A + B + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{C} + D)$$

EXAMPLE 2.28. Solve by QM method—

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$$

Solution: Step 1: Arranging all minterms index wise–

$f(A, B, C, D) = \Sigma m$ (0000, 0001, 0010, 0011, 0101, 0111, 1000, 1001, 1011 1110

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

Index0 Index1 Index1 Index2 Index2 Index3 Index1 Index2Index3 3

Index	Minterms	Variable				
		A	*B*	*C*	*D*	
0	0	0	0	0	0	✓
1	1	0	0	0	1	✓
	2	0	0	1	0	✓
	8	1	0	0	0	✓
2	3	0	0	1	1	✓
	5	0	1	0	1	✓
	9	1	0	0	1	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
	14	1	1	1	0	—Unchecked Unticked $ABC\overline{D}$

Index	Minterms (Grouping of 2 is)	Variables				
		A	*B*	*C*	*D*	
0	0,1	0	0	0	—	✓
	0, 2	0	0	—	0	✓
	0, 8	—	0	0	0	✓
1	1, 3	0	0	—	1	✓
	1, 5	0	—	0	1	✓
	1, 9	—	0	0	1	✓
	2, 3	0	0	1	—	✓
	8, 9	1	0	0	—	✓
2	3, 7	0	—	1	1	✓
	3, 11	—	0	1	1	✓
3	5, 7	0	1	—	1	✓
	9, 11	1	0	—	1	✓

Index	Minterms (Grouping of 4 1's)	A	B	C	D
		Variables			
0	0, 1, 2, 3	0	0	—	—
	0, 1, 8, 9	—	0	0	—
	0, 2, 1, 3	0	0	—	—
	0, 8, 1, 9	—	0	0	—
1	1, 3, 5, 7	0	—	—	1
	1, 3, 9, 11	—	0	—	1
	1, 5, 3, 7	0	—	—	1
	1, 9, 3, 11	—	0	—	1

No further grouping

Hence $f = \bar{A}\,\bar{B} + \bar{B}\bar{C} + \bar{A}D + \bar{B}D + ABC\bar{D}$

PI Table :

PI	Decimal Numbers	0	1	2	3	5	7	8	9	11
$ABC\bar{D}$	14									
$\bar{A}\bar{B}$	0, 1, 2, 3	×	×	⊗	×					
$\bar{B}\bar{C}$	0, 1, 8, 9	×	×					⊗	×	
$\bar{A}D$	1, 3, 5, 7		×		×	⊗	⊗			
$\bar{B}D$	1, 3, 9, 11		×		×					⊗
				✓		✓	✓	✓		✓

2 is coming only in $\bar{A}\,\bar{B}$
5, 7 is coming only is $\bar{A}D$
8 is coming only in $\bar{B}\bar{C}$
11 is coming only in $\bar{B}D$
14 is coming only in $ABC\bar{D}$

(0, 1, 2, 3)
(1, 3, 5, 7)
(0, 1, 8, 9)
(1, 3, 9, 11)
(14)
⇓

Hence 0, 1, 2, 3, 5, 7, 8, 9, 11, 14

$$f = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}D + \bar{B}D + ABC\bar{D}$$

EXAMPLE 2.29. $f(A, B, C, D) = \Sigma m(1, 3, 7, 11, 15) + d(0, 2, 5)$: QM match

Solution:

Σm	(1,	3,	7,	11,	15,	+ d	(0,	2,	5)
	↓	↓	↓	↓	↓		↓	↓	↓
	0001	0011	0111	1011	1111		0000	0010	0101
	1	2	3	3	4		0	1	2

Index	Minterms	Variables				
0	0*	0	0	0	0	✓
1	1	0	0	0	1	✓
	2*	0	0	1	0	✓
2	3	0	0	1	1	✓
	5*	0	1	0	1	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
4	15	1	1	1	1	✓

Index	Minterms	Variables				
		A	B	C	D	
0	0*, 1	0	0	0	—	✓
	0*, 2*	0	0	—	0	✓
1	1, 3	0	0	—	1	✓
	1, 5	0	—	0	1	✓
	2*, 3	0	0	1	—	✓
2	3, 7	0	—	1	1	✓
	3, 11	—	0	1	1	✓
	5*,7	0	1	—	1	✓
3	7, 15	—	1	1	1	✓
	11, 15	1	—	1	1	✓

Index	Minterms	Variables				
0	0*, 1, 2, * 3	0	0	—	—	$\bar{A}\bar{B}$
	0*, 2*, 1, 3	0	0	—	—	
1	1, 3, 5*, 7	0	—	—	1	$\bar{A}D$
	1, 5*, 3, 7	0	—	—	1	
2	3, 7, 11, 1 5	—	—	1	1	
	3, 11, 7, 15	—	—	1	1	CD

PI Table

		Decimal	All minterms							
			0*	1	2*	3	5*	7	11	15
	$\bar{A}\bar{B}$	0, 1, 2, 3	⊗	×	⊗	×				
	$\bar{A}D$	1, 3, 5, 7		×		×	⊗	×		
EPI	CD	3, 7, 11, 15				×		×	⊗	⊗
			✓		✓		✓		✓	✓

11 & 15 are covered by CD and rest all are covered by $\bar{A}D$. Hence, $y = \bar{A}D + CD$.