6.4. GREIBACH NORMAL FORM ✓

Definition For every context free language L without $\in$, there exist a grammar in which every production is of the form $A \to aV$, where 'A' is a variable, 'a' is exactly one terminal and 'V' is the string of none or more variables, clearly $V \in V_n^*$.

"In other words if every production of the context free grammar is of the form $A \to aV/a$, then it is in Greibach Normal Form."

Greibach normal form will be used to construct a push down atuomata that recognize the language generated by a context free grammar.

To convert a grammar to GNF we start with a production in which the left side has a higher numbered variable than first variable in the right side and make replacements in right side.

6.4.1. Left Recursion and its Removal

A CFG containing rules of the form

$$A \to A\alpha/\beta$$

is called left recursive in A. The language geneated by such rules is of the form $A \overset{*}{\Rightarrow} \beta\alpha^n$. If we replace the rules $A \to A\alpha/\beta$ by

$$A \to \beta A'/\beta$$

$$A' \to \alpha A'/\alpha$$

where A' is a new variable, then language generated by A is the same while no left-recursive A-rules are used in the derivation.

GNF

Que: Construct a grammar in GNF.

$S \to AA \mid a$

$A \to SS \mid b$

Sol:

Put $S = A_1$ & $A = A_2$

then

$A_1 = A_2 A_2 \mid a$ —— (1)

$A_2 = A_1 A_1 \mid b$ —— (2)

Putting the value of A_1 in (2)

$A_2 = A_2 A_2 A_1 \mid a A_1 \mid b$

($\alpha = A_2 A_1$, $\beta = a A_1$, $\beta = b$)

Using the ~~pumpi~~ lemma

$A = \beta$	$A = \beta Z$
$Z = \alpha$	$Z = \alpha Z$

$A_2 = a A_1 \mid b \mid a A_1 Z_2 \mid b Z_2$

$Z_2 = A_2 A_1 \mid A_2 A_1 Z_2$

from (1) $\Rightarrow$ $A_1 = a A_1 A_2 \mid b A_2 \mid a A_1 Z_2 A_2 \mid b Z_2 A_2 \mid a$

Now, Z_2 is not in the form of $a\alpha$.

then put the value of A_2 in Z_2

$Z_2 = a A_1 A_1 \mid b A_1 \mid a A_1 Z_2 A_1 \mid b Z_2 A_1$

$Z_2 = a A_1 A_1 Z_2 \mid b A_1 Z_2 \mid a A_1 Z_2 A_1 Z_2 \mid b Z_2 A_1 Z_2$

Q.

$A \to BS \mid b$
$B \to SA \mid a$

Put $S = A_1$, $A = A_2$, $B = A_3$

then

$A_1 = A_2 A_3$ — (1)

$A_2 = A_3 A_1 \mid b$ — (2)

$A_3 = A_1 A_2 \mid a$ — (3)

put the value A_1 in (3)

$A_3 = A_2 A_3 A_2 \mid a$ — (4)

Put the value A_2 in (4)

$A_3 = A_3 \underbrace{A_1 A_3 A_2}_{\alpha} \mid \underbrace{b A_3 A_2}_{\beta} \mid \underbrace{a}_{\beta}$

using lema

$A \to \beta$	$A \to \beta Z$
$Z \to \alpha$	$Z \to \alpha Z$

$A_3 = bA_3A_2 \mid a \mid bA_3A_2Z_3 \mid aZ_3$ — (5)

$Z_3 = A_1A_3A_2 \mid A_1A_3A_2Z_3$ — (6)

(2) ⇒ $A_2 = bA_3A_2A_1 \mid aA_1 \mid bA_3A_2Z_3A_1 \mid aZ_3A_1 \mid b$

(1) ⇒ $A_1 \to bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2Z_3A_1A_3 \mid aZ_3A_1A_3 \mid bA_3$

(6) ⇒ $Z_3 = bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_2Z_3A_1A_3A_3A_2 \mid$
$= aZ_3A_1A_3A_3A_2 \mid bA_3A_3A_2 \mid$
$= bA_3A_2A_1A_3A_3A_2Z_3 \mid aA_1A_3A_3A_2Z_3 \mid bA_3A_2Z_3A_1A_3A_3A_2Z_3 \mid$
$= aZ_3A_1A_3A_3A_2Z_3 \mid bA_3A_3A_2Z_3$

Example 6.18. Convert the grammar.

$$S \rightarrow AB \mid BC$$
$$A \rightarrow aB \mid bA \mid a$$
$$B \rightarrow bB \mid cC \mid b$$
$$C \rightarrow c$$

into GNF.

Solution. Here the production $S \rightarrow AB \mid BC$ is not in GNF. On applying the substitution rule we immediately get equivalent grammar.

$$S \rightarrow aBB \mid bAB \mid aB \mid bBC \mid cCC \mid bC$$

which is in GNF.

CamScanner

Example 6.19. Convert the grammar.

$$S \rightarrow abaSa \mid aba$$

into GNF.

Solution. If we introduce new variables A and B and productions as $A \rightarrow a$, $B \rightarrow b$ and substitute into given grammar as

$$S \rightarrow aBASA \mid aBA$$

$$A \rightarrow a,\ B \rightarrow b$$

which is in GNF.

which is in Greibach ...

Example 6.22. Convert the following grammar

$$S \to abSb/aa$$

in to GNF.

Solution. Here we can use a method similar to the one introduced in the construction of Chomsky normal form. We introduce new variables A and B that are essentially synonyms for a and b, respectively, substituting for the terminals with their associated variables leads to the equivalent grammar.

$$S \to aBSB/aA$$
$$A \to a$$
$$B \to b$$

which is in GNF.

Example 6.23. Convert the following grammar into GNF

$$S \to aAS$$
$$S \to a$$
$$A \to SbA$$
$$A \to SS$$
$$A \to ba.$$

Solution. Let us introduced new non-terminals as B, C and D, then GNF will be

$$A \to aBC$$
$$A \to aBS$$
$$A \to aC$$
$$A \to aS$$
$$A \to bD$$
$$B \to aBCS$$
$$B \to aBSS$$
$$B \to aCS$$
$$B \to aSS$$