Example 6.11. Consider the following grammar

$$S \rightarrow a/Xb/aYa$$
$$X \rightarrow Y/\in$$
$$Y \rightarrow b/X$$

eliminate $\in$-productions.

Solution. Here $X \rightarrow \in$ is $\in$-productions. So this problem can also solved in similar fashion and grammar without $\in$-productions will be

$$S \rightarrow a/Xb/aYa/b/aa$$
$$X \rightarrow Y$$
$$Y \rightarrow b/X.$$

Example 6.12. Design a CFG for regular expression $r = (a + b)^*\ bb\ (a + b)^*$, which is free from $\in$-productions.

Solution. Let CFG G be required context free grammar with $\in$-productions

$$S \rightarrow XY$$
$$X \rightarrow Zb$$
$$Y \rightarrow bW$$
$$Z \rightarrow AB$$
$$W \rightarrow Z$$
$$A \rightarrow aA/bA/\in$$
$$B \rightarrow Ba/Bb/\in$$

Finally here $A \rightarrow \in$ and $B \rightarrow \in$ are $\in$-productions and A, B are nullable non-terminals.

The CFG without $\in$-production can be achieved from G in similar fashion, as above discussed. Let it be G', then G' is

$$S \rightarrow XY$$
$$X \rightarrow Zb/b$$
$$Y \rightarrow bW/b$$
$$Z \rightarrow AB/A/B$$
$$W \rightarrow Z$$
$$A \rightarrow aA/bA/a/b$$
$$B \rightarrow Ba/Bb/a/b$$

Because $\in$ was not generated by G, the new CFG G' generates exactly the same language.

6.3. CHOMSKY NORMAL FORM

Definition If a CFG has only production of the form
Non-terminals → string of exactly two non-terminals or of the form
Non-terminals → one terminal
is said to be chomsky normal form or CNF.

6.3.1. Conversion to Chomsky Normal Form

"Conversion of the grammar to CNF can be understood by the following example"

Let us conside a grammar

$G = (\{S, A, B\}, \{a, b\}, P, S)$, where S is the start symbol and P is given by

$$S \rightarrow bA/aB$$
$$A \rightarrow bAA/aS/a$$
$$B \rightarrow aBB/bS/a$$

Now let us find an equivalent chomsky normal form of it.

As we know that right side in CNF either contains two non-terminals or one terminal. So it is clear that in the first production above, we have to replace terminal 'b' by a non-terminal say C_b and 'a' by C_a, hence grammar becomes

$$S \rightarrow C_bA/C_aB$$
$$A \rightarrow C_bAA/C_aS/a$$
$$B \rightarrow C_aBB/C_bS/b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$

Now first rule is in CNF. In second rule C_bAA is not in CNF. So we can replace AA by a non-terminal say D and similarly BB by E, we obtain

$$S \rightarrow C_bA/C_aB$$
$$A \rightarrow C_bD/C_aS/a$$
$$B \rightarrow C_aE/C_bS/b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$
$$D \rightarrow AA$$
$$E \rightarrow BB$$

The above grammar is equivalent Chomsky normal form of the given grammar.

Example 6.13. Change the following grammar in to CNF

$$S \rightarrow 1A/0B$$
$$A \rightarrow 1AA/0S/0$$
$$B \rightarrow 0BB/1$$

Solution. We know that in CNF, either two non-terminal or one terminal is on the right side.

Therefore replace AA by C_a and BB by C_b, 1 by C_1, 0 by C_0, we get

$$S \rightarrow C_1A/C_0B$$
$$A \rightarrow C_1C_a/C_0S/0$$

$$B. \rightarrow C_0C_b/1$$
$$C_1 \rightarrow 1$$
$$C_0 \rightarrow 0$$
$$C_a \rightarrow AA$$
$$C_b \rightarrow BB$$

while the CNF form.

Example 6.14. Change the following grammar in to CNF

$$G = (\{S\}, \{a, b, c\}, \{S \rightarrow a/b/CSS\}, S\}.$$

Solution. Here productions are

$$S \rightarrow a$$
$$S \rightarrow b$$
$$S \rightarrow CSS$$

Replace SS by D, we get

$$S \rightarrow a$$
$$S \rightarrow b$$
$$S \rightarrow CD$$
$$D \rightarrow SS$$

which is in CNF form.

Example 6.15. Change the following grammar in to CNF

$$S \rightarrow abSb/a/aAb$$
$$A \rightarrow bS/aAAb.$$

Solution. Replace aA by B_a and Ab by B_b, we get

$$S \rightarrow abSb/a/aB_b$$
$$A \rightarrow bS/B_aB_b$$
$$B_a \rightarrow aA$$
$$B_b \rightarrow Ab$$

Now replace ab by C and Sb by D, we get

$$S \rightarrow CD/a/aB_b$$
$$A \rightarrow bS/B_aB_b$$
$$B_a \rightarrow aA$$
$$B_b \rightarrow aA$$
$$B_b \rightarrow Ab$$
$$C \rightarrow ab$$
$$D \rightarrow Sb$$

Now replace a by X_a and b by X_b, we get

$$S \rightarrow CD/a/X_aB_b$$
$$A \rightarrow X_bS/B_aB_b$$

$$B_a \rightarrow X_aA$$
$$B_b \rightarrow X_aB$$
$$C \rightarrow AX_b$$
$$D \rightarrow SX_b$$
$$X_a \rightarrow a$$
$$X_b \rightarrow b.$$

It is required CNF form.

Example 6.16. Convert CFG which is given below in to CNF form.

$$S \rightarrow bA/aB$$
$$A \rightarrow bAA/aS/a$$
$$B \rightarrow aBB/bS/b.$$

Solution. Let us replace b by C_b and a by C_a, then CFG becomes

$$S \rightarrow C_bA/C_aB$$
$$A \rightarrow C_bAA/C_aS/a$$
$$B \rightarrow C_aBB/C_bS/b$$
$$C_b \rightarrow b$$
$$C_a \rightarrow a$$

Now let us replace C_bA by D and C_aB by E then grammar becomes as follows :

$$S \rightarrow C_bA/C_aB$$
$$A \rightarrow DA/C_aS/a$$
$$B \rightarrow EB/C_bS/b$$
$$C_b \rightarrow b$$
$$C_a \rightarrow a$$
$$D \rightarrow C_bA$$
$$E \rightarrow C_aB$$

Now every production of the grammar is in the CNF form.

... and convert