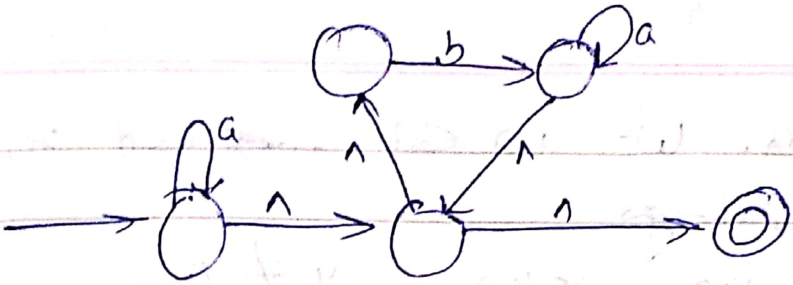
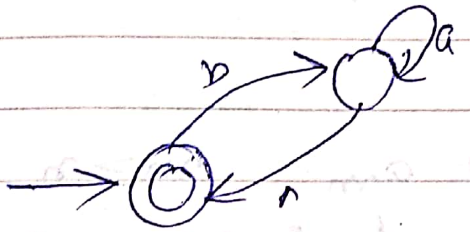
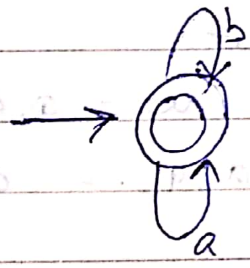






Pumping Lemma for Regular Sets :→

We give a necessary condition for an input string to belong to a regular set. The result is called pumping lemma as it gives a method of pumping (generating) many input strings from a given string. As pumping lemma gives a necessary condition, it can be used to show that certain sets are not regular.

Theorem :- Pumping lemma :→

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton with n states. Let L be the regular set accepted by M. Let $w \in L$ ~~.~~ and x, y, z are two strings.

Such that $w = xyz$, $y \neq \wedge$

and $xy^iz \in L$ for each $i \geq 0$

Proof : Let

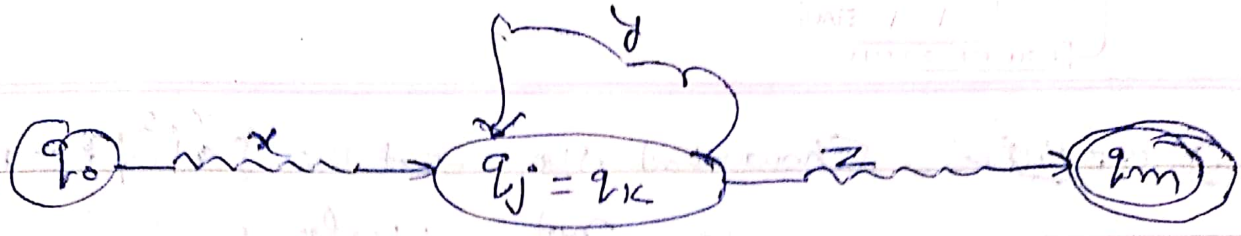
$$w = a_1 a_2 \ldots a_m \quad m \geq n$$

$$\delta(q_0, a_1 a_2 \ldots a_i) = q_i \text{ for } i = 1, 2, \ldots m;$$

$$Q_1 = \{q_0, q_1, \ldots q_m\}$$

That is Q_1 is the sequence of states in the path with path value $w = a_1 a_2 \ldots a_m$. As there are only n distinct states, at least two states in Q_1 must coincide. Among various pairs of repeated states, we take the first pair. Let us take them as q_j and q_k $(q_j = q_k)$. Then j and k satisfy the condition $0 \leq j < k \leq n$

The string w can be decomposed into three substrings $a_1 a_2 \ldots a_j$, $a_{j+1} \ldots a_k$ and $a_{k+1} \ldots a_m$. Let x, y, z denote these strings $a_1 a_2 \ldots a_j$, $a_{j+1} \ldots a_k$ and $a_{k+1} \ldots a_m$ respectively. As $k \leq n$, $|xy| \leq n$ and $w = xyz$. The path with path value w in the transition diagram of M is shown below.



Here $xy^iz \in L$, As every state in Q_1 is obtained by applying an input symbol $y \neq \Lambda$

The decomposition is valid only for strings of length greater than or equal to the number of states.

Application of Pumping Lemma :-

This theorem can be used to prove that certain sets are not regular.

Step 1. Assume L is a regular. Let n be the number of states in the corresponding FA.

Step 2. Choose a string w such that $|w| \geq n$ use pumping lemma to write $w = xyz$ with $|xy| \leq n$ and $|y| > 0$

Step 3. Find a suitable integer i such that $xy^iz \notin L$. This contradicts our assumption hence L is not regular.

Que - 4.18 Show that the set $L = \{a^{i^2} \mid i \geq 1\}$ is not regular.

Sol:-

$L = \{a^{i^2} \mid i \geq 1\}$
$= \{a^2, a^4, a^6, \ldots\}$
$= \{aa, aaaa, aaaaaa, \ldots\}$

Let $w = xy^iz$
where $x = \wedge$
$y^i = a^i$
$z = a$

then $w = a^i a$

for $i=1$ $w = aa \in L$

for $i=2$ $w = a^2 a = aaa \notin L$

Hence we can say a^{i^2} are not regular

Que 4.19 Show that $L = \{a^p \mid p \text{ is a prime}\}$ is not regular.

Sol:-

$L = \{a^p \mid p \text{ is prime}\}$
$= \{a^2, a^3, a^5, a^7, \ldots\}$
$= \{aa, aaa, aaaaa, aaaaaaa, \ldots\}$

Let $w = xy^iz$
where
$x = \wedge$
$y = a$

then $w = a^i a$

for $i=2 \Rightarrow$ $w = a^2 a$ = aaa $\in L$

for $i=3 \Rightarrow$ $w = a^3 a$ = aaaa $\notin L$

Hence we can say $a^p \mid p$ is prime are not regular.

x ———— x

Que (4.20) Show that $L = \{0^i 1^i \mid i \geq 1\}$ is not regular.

Sol:-
$L = \{0^i 1^i \mid i \geq 1\}$

$L = \{01, 0011, 000111, \ldots\}$

Let $w = xy^i z$
where $x = \Lambda$
$y = 0$
$z = 1$

then $w = 0^i 1$

for $i=1$ $w = 0^1 1 = 01 \in L$

for $i=2$ $w = 0^2 1 = 001 \notin L$

Hence we can say $0^i 1^i$ are not regular.

x ———— x

Que (4.21) Show that $L = \{ww \mid w \in (a,b)^*\}$ is not regular

Sol:-

We suppose L is regular and get a contradiction. Let n be a no of states in FA accepting L.

let us consider $L = ww = a^n b a^n b \mid n \geq 1$

$L = \{ abab, aabaab, aaabaaab, \ldots$

let $K = x y^i z$

where

$x = a$

$y = b$

$z = \wedge$

then $K = ab^i$

for $i = 1 \Rightarrow K = ab \notin L$

$i = 2 \Rightarrow K = abb \notin L$

Hence we can say $L = ww = (a^n b a^n b)$ are not regular.

x x