Algebraic method using Arden's Theorem

This is used to find the r.e recognised by transition system:

The following assumptions are made regarding the transition system:

(i) The transition graph does not have Λ-moves.

(ii) It has only one initial state, say v_1

(iii) Its vertices are $v_1 \,---\, v_n$

(iv) v_i is the r.e representing the set of strings accepted by the system even though v_i is final state.

Ques: Consider the transition system as shown in below diagram. Prove that the strings recognised are $(a + a(b+aa)^* b)^* a(b+aa)^* a$

Sol:

$$q_1 = q_1 a + q_2 b + \Lambda \quad \text{——— } (1)$$

$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{——— } (2)$$

$$q_3 = q_2 a \quad \text{——— } (3)$$

By (2) $\Rightarrow$

$$q_2 = q_1 a + q_2 b + q_3 a$$

$$= q_1 a + q_2 b + q_2 aa$$

$$q_2 = q_1 a + q_2 (b + aa)$$

$$R = Q + R \quad P$$

$R = Q + PR$ then $R = QP^*$

$$q_2 = q_1 a (b+aa)^* \quad \text{—— (4)}$$

Put q_2 value in q_1 or by (1) $\Rightarrow$

$$q_1 = q_1 a + q_1 a (b+aa)^* b + \wedge$$

$$q_1 = q_1 (a + a(b+aa)^* b) + \wedge$$

$$R = RP + Q$$

then $R = QP^*$

$$q_1 = \wedge (a + a(b+aa)^* b)^* \quad \text{—— (5)}$$

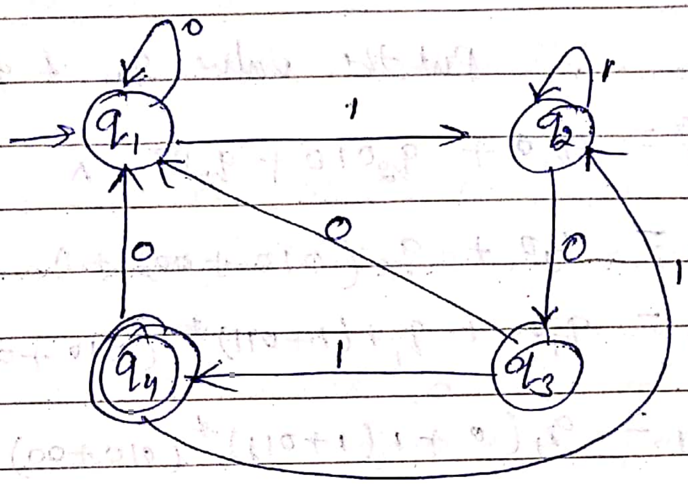
put the value of q_1 is (4)

$$q_2 = (a + a(b+aa)^* b)^* a (b+aa)^*$$

by (3) $\Rightarrow$ $q_3 = (a + a(b+aa)^* b)^* a (b+aa)^* a$

Hence it is prove.

Ques:- Find the r.e. corresponding the fig.



$q_1 = q_1 0 + q_4 0 + q_3 0 + \Lambda$ —— (1)

$q_2 = q_1 1 + q_2 1 + q_4 1$ —— (2)

$q_3 = q_2 0$ —— (3)

$q_4 = q_3 1$ —— (4)

By (4) $\Rightarrow$ $q_4 = q_2 0 1$ —— (5)

by (2) $\Rightarrow$ $q_2 = q_1 1 + q_2 1 + q_2 011$

$q_2 = q_1 1 + q_2 (1 + 011)$

$R = Q + RP$

$R = QP^*$

then

$q_2 = (q_1 1)(1 + 011)^*$ —— (6)

By (1) $\Rightarrow$ $q_1 = q_0 0 + q_1 0 + q_3 0 + \Lambda$

Put the value q_4 & q_3

$q_1 = q_1 0 + q_2 010 + q_2 00 + \Lambda$

$q_1 = q_1 0 + q_2 (010 + 00) + \Lambda$

$q_1 = q_1 0 + q_1 1 (1+011)^* (010+00) + \Lambda$

$q_1 = q_1 (0 + 1(1+011)^* (010+00)) + \Lambda$

$R = RP + Q$

$R = QP^*$

then

$q_1 = \Lambda(0 + 1(1+011)^* (010+00))^*$

then By (4) $\Rightarrow$

$q_4 = q_3 1$

$= q_2 01$

$= q_1 1 (1+011)^* 01$

$q_4 = (0 + 1(1+011)^* (00+010))^* (1(1+011)^* 01)$

this is the final state.

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