(*) Regular Sets and Regular Grammars.

Regular Expressions :->

1). Any terminal Symbol (i.e. an element of Σ), Λ and ϕ are regular expressions. if used with a relation $R \cdot \Lambda$ then it is 'R'

2) The union of two regular expressions R_1 and R_2 written as $R_1 + R_2$ is also regular expressions.

3). The concatenation of two regular expression, R_1 and R_2 written as $R_1 R_2$, also regular expression,

4). The iteration (or closure) of a regular expression R written as R^* is also regular expression.

5). If R is regular expression the (R) is also a regular expression.

Note :-> In evaluating a regular expression we perform iteration first, then concatenation and finally union, As same as (exponentiation, multiplication & addition.)

Ques:-

Eg:- (a) $\{101\}$,
(b) $\{abba\}$
(c) $\{01, 10\}$
(d) $\{\Lambda, ab\}$
(e) $\{abb, a, b, bba\}$
(f) $\{\Lambda, 0, 00, 000, ----\}$
(g) $\{1, 11, 111, ----\}$

Sol:- (a) 1 & 0 are two strings, then it represented by 101

(b) $\{abba\}$ represented $abba$

(c) $\{01, 10\}$ represented $01 + 10$

(d) The set $\{\Lambda, ab\}$ is represented by $\Lambda + ab$

(e) $\{abb, a, b, bba\}$ represented $abb + a + b + bba$

(f) As $\{\Lambda, 0, 00, 000, ----\}$ is 0^*

(g) As $\{1, 11, 111 ----\}$ $1(1)^*$

Identities for Regular Expressions.

$I_1 \rightarrow \phi + R = R$

$I_2 \rightarrow \phi R = R\phi = \phi$

$I_3 \rightarrow \Lambda R = \Lambda R = R$

$I_4 \rightarrow \Lambda^* = \Lambda$ & $\phi^* = \Lambda$

$I_5 \rightarrow R + R = R$

$I_6 \rightarrow R^* R^* = R^*$

$I_7 \rightarrow R R^* = R^* R = R^*$

$I_8 \rightarrow (R^*)^* = R^*$

$I_9 \rightarrow \Lambda + R R^* = R^* = \Lambda + R^* R$

$I_{10} \rightarrow (PQ)^* P = P(QP)^*$

$I_{11} \rightarrow (P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

$I_{12} \rightarrow (P+Q)R = PR + QR$ and $R(P+Q) = RP + RQ$

Arden's Theorem :-

$$\boxed{R = Q + RP} \longrightarrow (1)$$

has a unique soln. (i.e., one and only) $R = QP^*$

Proof :- By (1) of R.H.S.

$= Q + (QP^*)P$

$= Q(\Lambda + P^* P)$

$= QP^*$ By I_9.

Hence. $R = QP^*$

In other words :-

$R = Q + RP$

$= Q + (Q + RP)P$ Put value of R

$= Q + QP + RPP$

$= Q + QP + RP^2$ we put value of R

$= Q + QP + (Q + RP)P^2$

$= Q + QP + QP^2 + RP^3$

$-$ $\longrightarrow$

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$= Q + QP + QP^2 + QP^3 + \text{———} QP^i + RP^{i+1}$

$= Q\underbrace{(\wedge + P + P^2 + P^3 + \text{———} P^i)}_{P^*} + RP^{i+1}$

$R = Q P^*$

Que :- Prove

$$(1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1) = 0^*1(0+10^*1)^*$$

Sol^n:

L.H.S.

$$= (1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$$

$$= (1+00^*1)(\Lambda + \underbrace{(0+10^*1)}_{P}{}^*\underbrace{(0+10^*1)}_{P})$$

$$= (1+00^*1)(\Lambda + (0+10^*1)^*) \quad \text{By } I_9$$

$$= (1+00^*1)(0+10^*1)^* \quad \text{By } I_9$$

$$= 1(\Lambda + 00^*)(0+10^*1)^*$$

$$= 10^*(0+10^*1)^* \quad \text{By } I_9$$

$$= 0^*1(0+10^*1)^*$$

$$= \text{R.H.S.}$$