Maxima methods for defuzzification: FoM, LoM and MoM

Maxima methods are quite simple but not as trivial as lambda cut methods. Maxima methods relies on the position of maximum membership of element at particular position in fuzzy set.

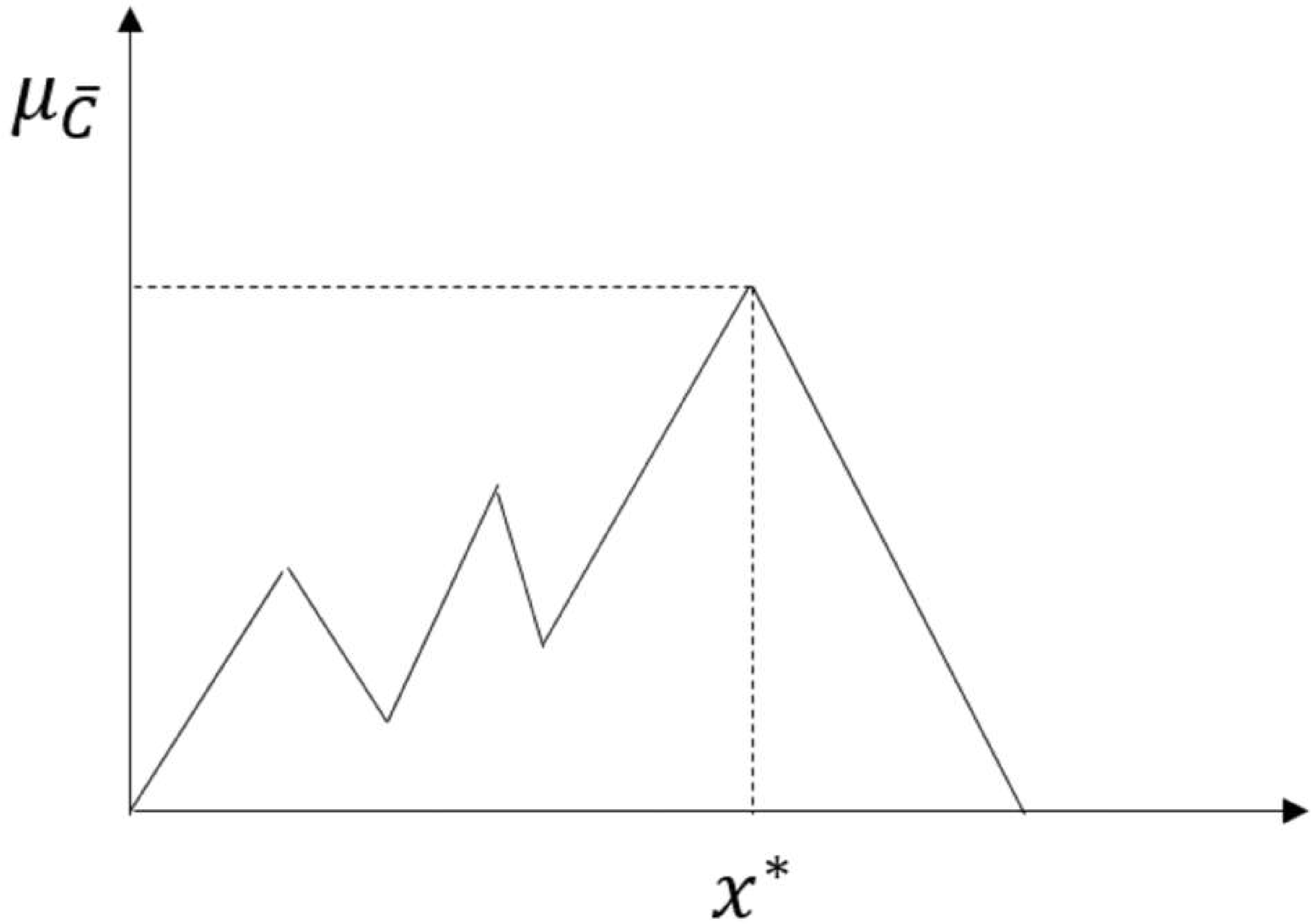
The set of methods under maxima methods we will be discussing here are:

- Height method
- First of maxima (FoM)
- Last of maxima (LoM)
- Mean of maxima (MoM)

Height method:

This method is based on **Max-membership principle**, and defined as follows.

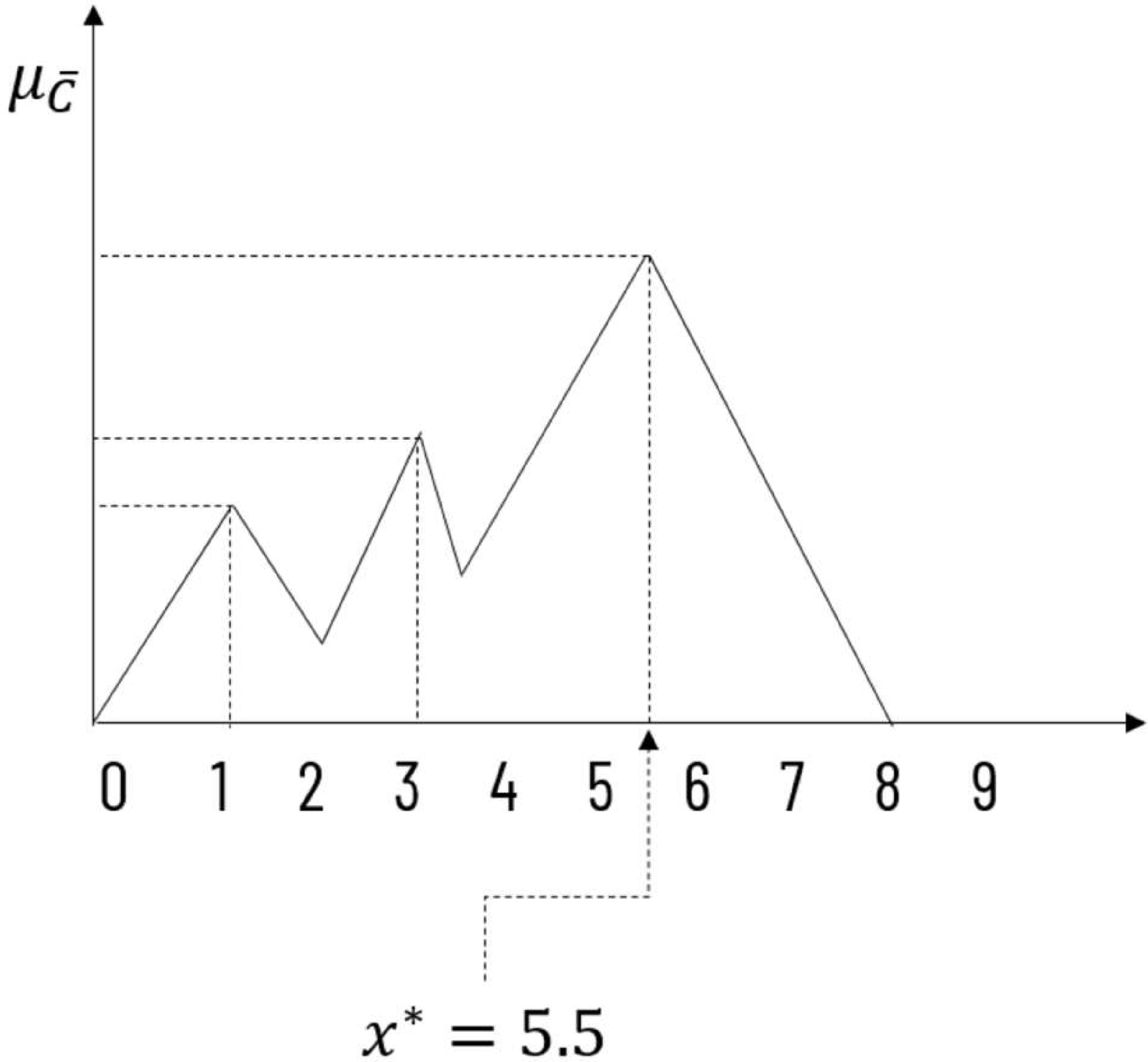
$$\mu_{\underline{C}}(x^*) \geq \mu_{\underline{C}}(x), \forall x \in X$$



Height method

Note: This method is applicable when **height is unique**.

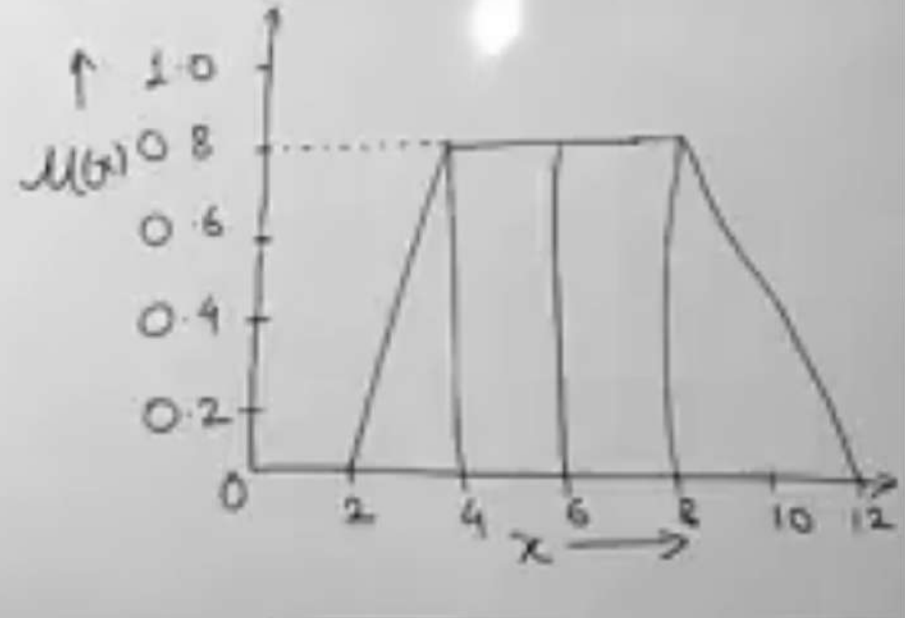
Example:



Example of height method

Maxima Methods

1. (I) First of Maxima (FOM)
2. (II) Last of Maxima (LOM)
3. (III) Mean of Maxima (MOM)



$$x^* = \frac{\sum_{x_i \in M} x_i}{|M|}$$

$M = \{ x \mid \mu_A(x) = \text{height of fuzzy set} \}$

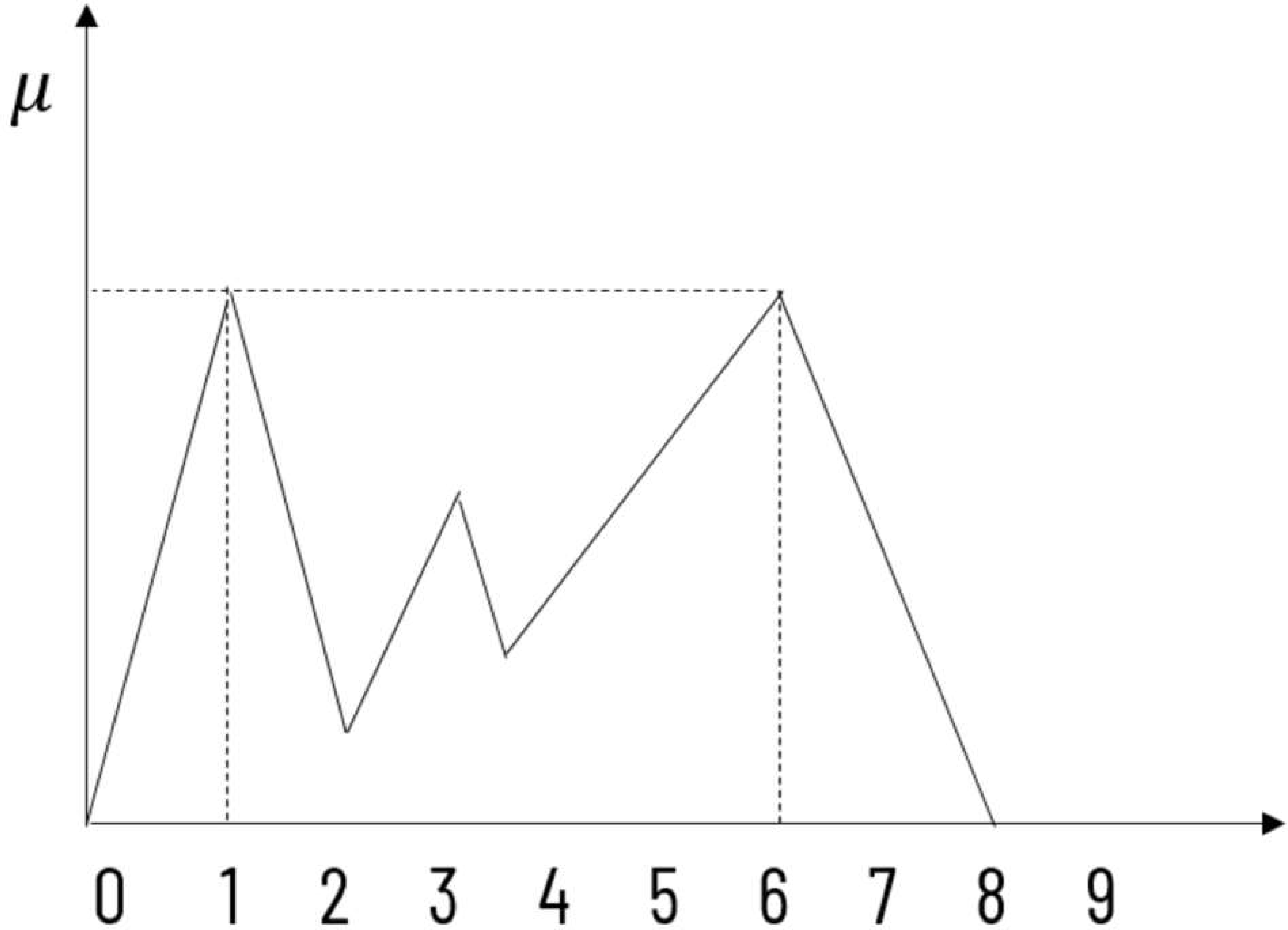
$|M|$ = Cardinality of set M

$x = 4, 6, 8$

$$x^* = \frac{4+6+8}{3}$$

$$\boxed{x^* = 6}$$

Find the defuzzification value for given fuzzy set

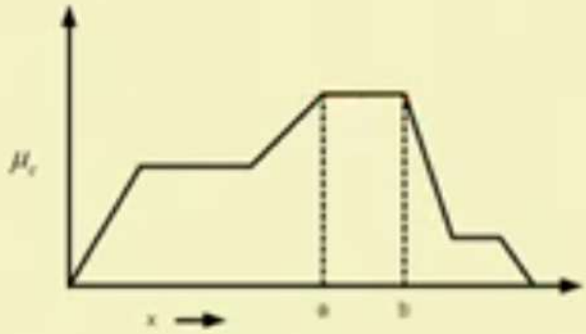


First of Maxima: $x^* = 1$

Last of Maxima: $x^* = 6$

MoM : Example 2

What is the crisp value of the fuzzy set using MoM in the following case?



$$x^* = \frac{a+b}{2}$$

Note:

- Thus, MoM is also synonymous to middle of maxima.
- MoM is also a general method of Height.

Lamda-Cut method. (Fuzzy sets)

- Fuzzy set A $\xrightarrow{\text{To}}$ Crisp set A_λ, $0 \leq \lambda \leq 1$)
- $A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$

Example :-

$A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$

$\lambda = 0.3$

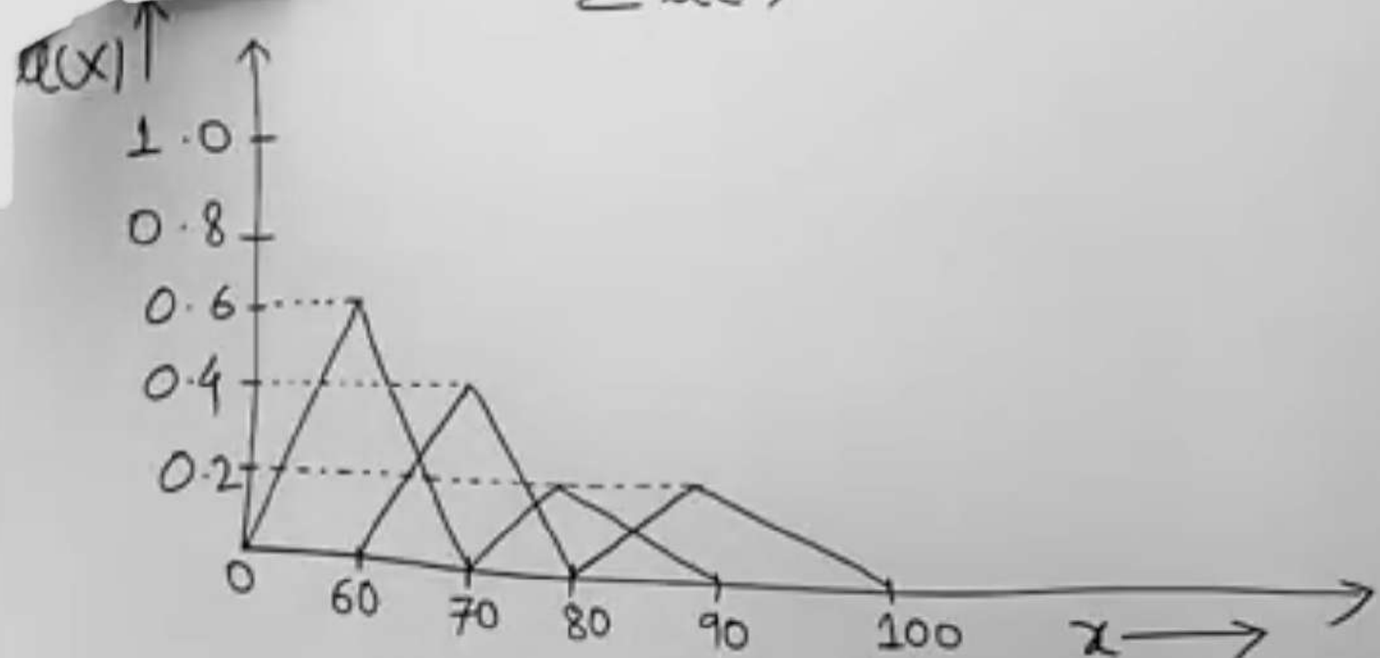
$A_{0.3} = \{(x_1, 0), (x_2, 1), (x_3, 1)\} = \{x_2, x_3\}$

$B = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$

$B_{0.7} = \{(y_1, 0), (y_2, 0), (y_3, 1)\} = \{y_3\}$

Weighted Average Method

$$x^* = \frac{\sum \mu(x) \cdot x}{\sum \mu(x)}$$



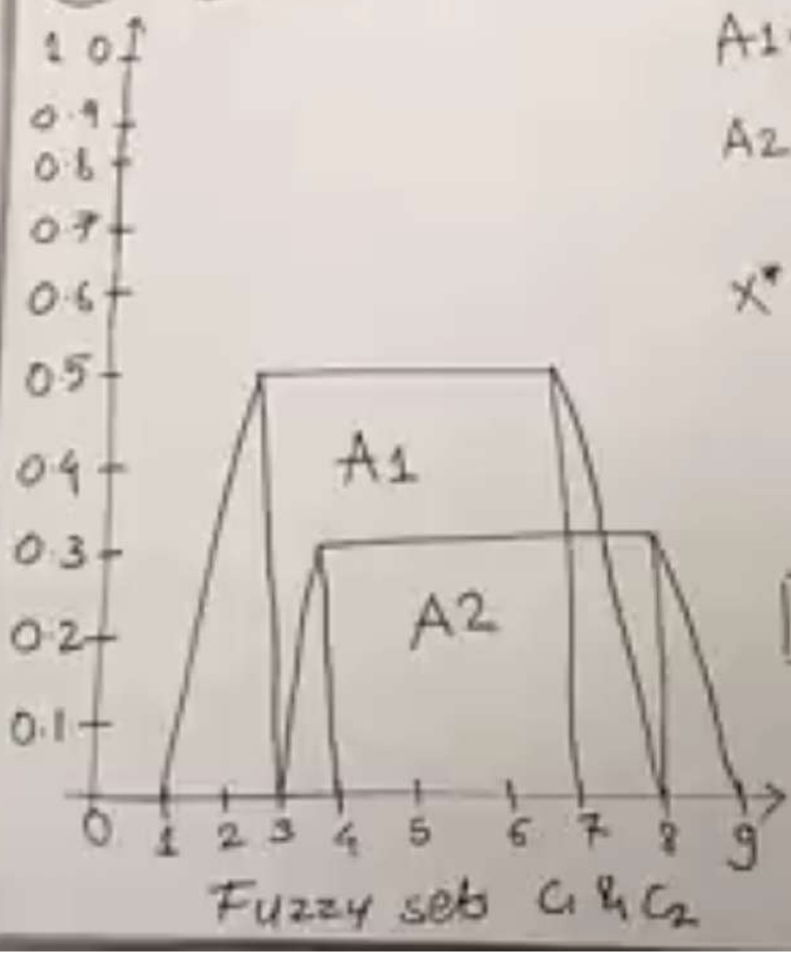
$$x^* = \frac{(60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2)}{0.6 + 0.4 + 0.2 + 0.2}$$

$$\boxed{x^* = 70}$$

Centroid Method

(I) Center of sum (COS)

$$x^* = \frac{\sum A \cdot x_c}{\sum A}$$



$$A1 = \frac{1}{2}\left[(8-1)+(7-3)\right] \times 0.5 = 2.75$$

$$A2 = \frac{1}{2}\left[(9-3)+(8-4)\right] \times 0.3 = 1.50$$

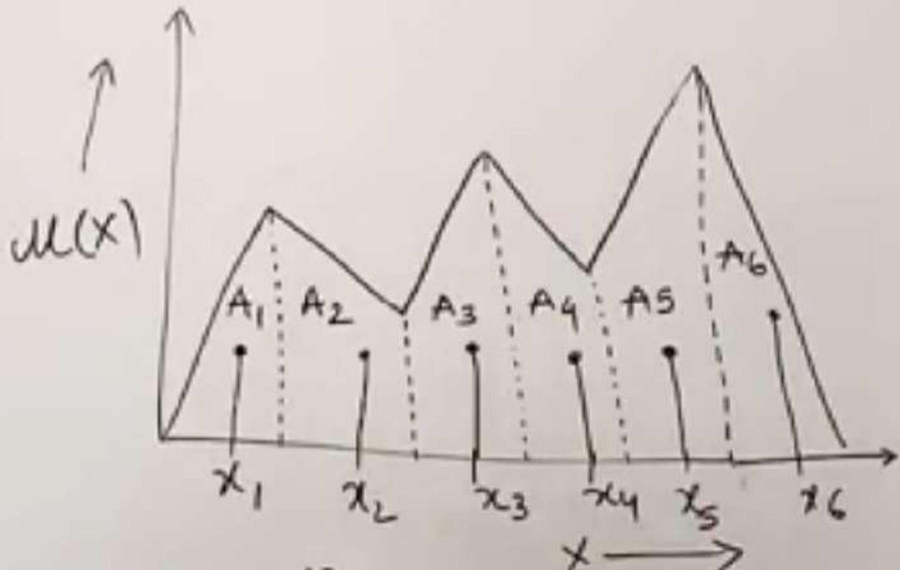
$$x^* = \frac{A_1 \cdot x_{c_1} + A_2 \cdot x_{c_2}}{A_1 + A_2}$$

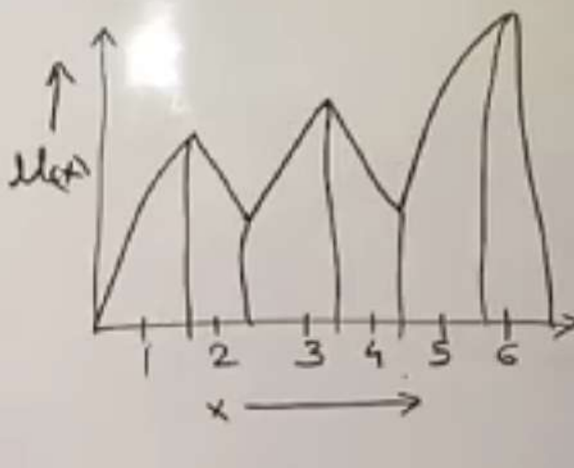
$$= \frac{2.75 \times 5 + 1.50 \times 6}{2.75 + 1.5}$$

$$\boxed{x^* = 5.35}$$

Centroid Method

(II) Center of Gravity (COG)





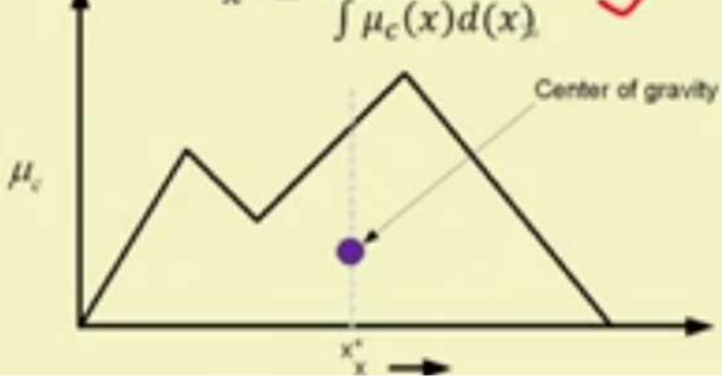
$$X^* = \frac{\sum_{i=1}^{n} x_i \cdot A_i}{\sum_{i=1}^{n} A_i}$$

Centroid method : CoG

1) The basic principle in CoG method is to find the point x where a vertical line would slice the aggregate into two equal masses.
2) Mathematically, the CoG can be expressed as follows :

$$x^* = \frac{\int x.\mu_c(x)d(x)}{\int \mu_c(x)d(x)}$$

3) Graphically,



CoG : A geometrical method of calculation

Steps:

1) Divide the entire region into a number of small regular regions (e.g. triangles, trapezoid, etc.)

2) Let A_i and x_i denotes the area and $c.g.$ of the i^{th} portion.
3) Then x^* according to CoG is

$$x^* = \frac{\sum_{i=1}^{n} x_i.(A_i)}{\sum_{i=1}^{n} A_i}$$

where n is the number of smaller geometrical components.

Centroid method: Centre of largest area

If the fuzzy set has two sub regions, then the centre of gravity of the sub region with the largest area can be used to calculate the defuzzified value.

Mathematically, $x^{*} = \frac{\int \mu_{c_m}(x).x' d(x)}{\int \mu_{c_m}(x)\, d(x)}$;

Here, C_m is the region with largest area, x' is the centre of gravity of C_m.

Graphically,