Fuzzy Control system

Design a controller to determine the wash time of a domestic washing machine. Assume the input is dirt & grease on cloths. Use three descriptors for input variables and five descriptor for output variable. Derive the set of rules for controller action and defuzzification. The design should be supported by figure wherever possible. Show that if the cloths are solid to a larger degree the wash time will be more and vice versa.

Steps to solve

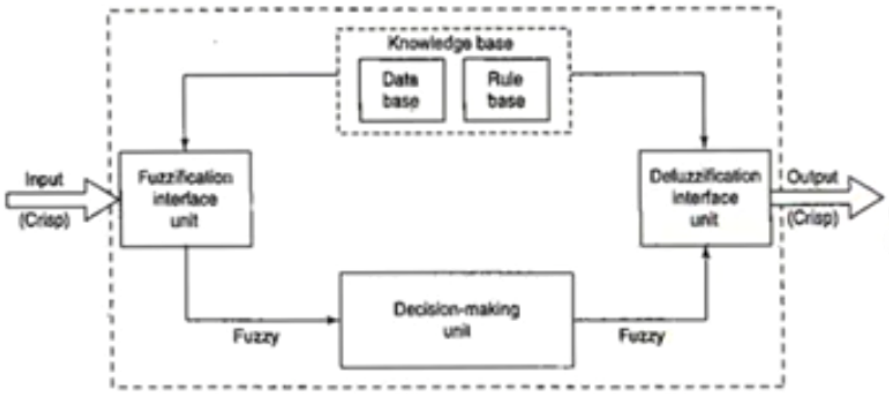
Step01: Identify input and output variables and decide descriptor for the same.

Step02: Define membership functions for each of input and output variables

Step03: Form a rule base

Step04: Rule Evaluation

Step05: Defuzzification



Step01: Identify input and output variables and decide descriptor for the same.

- Here inputs are “dirt” and “grease”. Assume they are in %
- Output is “wash time” measured in minute.

Descriptor for INPUT variable

Dirt

SD: Small dirt

MD: Medium dirt

LD: Large dirt

{SD, MD, LD}

Grease

NG: No Grease

MG: Medium Grease

LG: Large Grease

{NG, MG, LD}

Descriptor for OUTPUT variable

Wash Time

VS: Very Short

S: Short

M: Medium

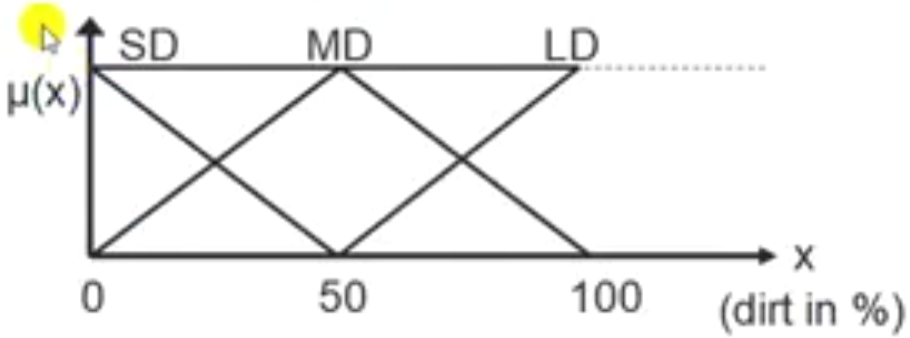
L: Large

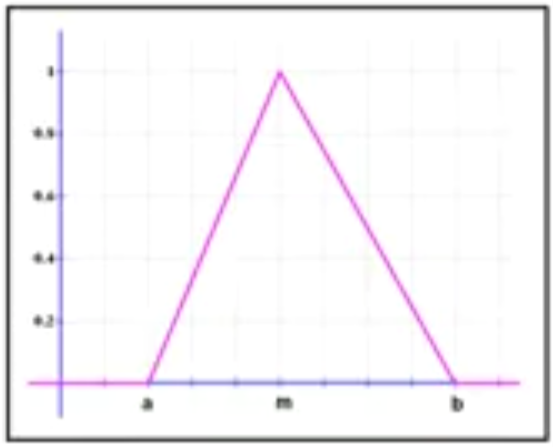
VL: Very Large

{VS, S, M, L, VL}

Step02: Define membership functions for each of input and output variables (We use triangular MF's)

(1) Membership function for dirt:



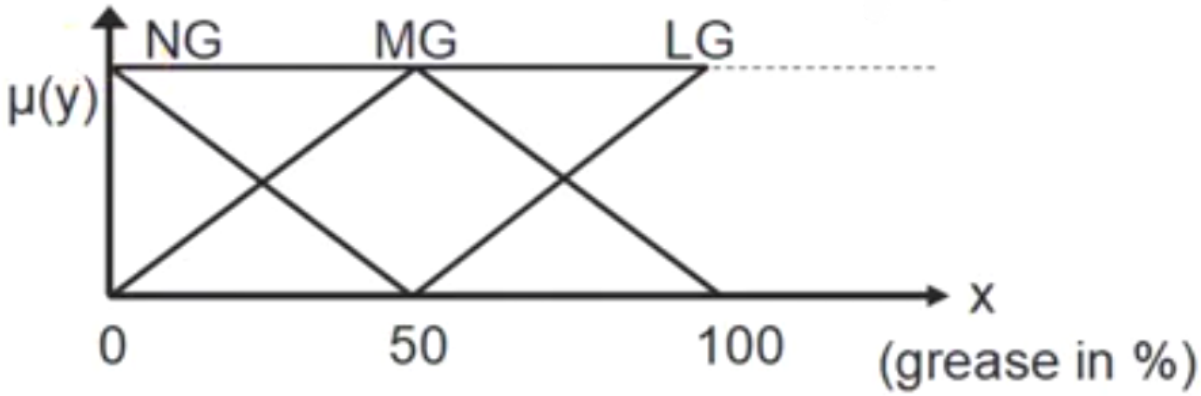


$$\mu_{SD}(x) = \frac{50 - x}{50}, 0 \le x \le 50$$

$$\mu_{MD}(x) = \begin{cases} \frac{x}{50}, & 0 \le x \le 50 \\ \frac{100 - x}{50}, & 50 \le x \le 100 \end{cases}$$

$$\mu_{LD}(x) = \frac{x - 50}{50}, 50 \le x \le 100$$

(2) Membership function for grease:

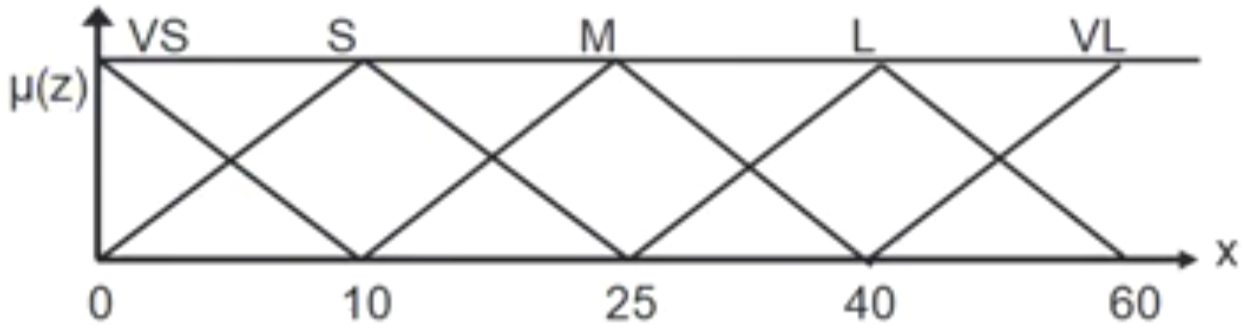


$$\mu_{NG}(y) = \frac{50 - y}{50}, 0 \le y \le 50$$

$$\mu_{MG}(y) = \begin{cases} \frac{y}{50}, & 0 \le y \le 50 \\ \frac{100 - y}{50}, & 50 \le y \le 100 \end{cases}$$

$$\mu_{LG}(y) = \frac{y - 50}{50}, 50 \le y \le 100$$

(3) Membership function for Wash time:



(wash time in min)

$$\mu_{VS}(z) = \frac{10 - z}{10}, 0 \le z \le 10$$

$$\mu_S(z) = \begin{cases} \frac{z}{10}, & 0 \le y \le 10 \\ \frac{25 - Z}{15}, & 10 \le z \le 25 \end{cases}$$

$$\mu_M(z) = \begin{cases} \frac{z - 10}{15}, & 10 \le Z \le 25 \\ \frac{40 - z}{15}, & 25 \le z \le 40 \end{cases}$$

$$\mu_L(z) = \begin{cases} \frac{z - 25}{15}, & 25 \le z \le 40 \\ \frac{60 - z}{20}, & 40 \le z \le 60 \end{cases}$$

$$\mu_{VL}(z) = \frac{z - 40}{20}, 40 \le z \le 60$$

Step03: Form a rule base

x \ y	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

Step04: Rule Evaluation

Assume Dirt = 60%, Grease= 70%

Dirt=60% maps two MFs of dirt Grease=70% maps 2 MFs

$$\mu_{MD}(x) = \frac{100 - x}{50} \mid \mu_{LD}(x) = \frac{x - 50}{50} \qquad \mu_{MG}(y) = \frac{100 - y}{50} \mid \mu_{LG}(y) = \frac{y - 50}{50}$$

Evaluate:

$$\mu_{MD}(60) = \frac{100 - 60}{50} = \frac{4}{5} \qquad \mu_{MG}(70) = \frac{100 - 70}{50} = \frac{3}{5}$$

$$\mu_{LD}(60) = \frac{60 - 50}{50} = \frac{1}{5} \qquad \mu_{LG}(70) = \frac{70 - 50}{50} = \frac{2}{5}$$

The above four equation leads to 4 rules need to evaluate:

1. Dirt is Medium and Grease is Medium
2. Dirt is Medium and Grease is Large
3. Dirt is Large and Grease is Medium
4. Dirt is Large and Grease is Large

Since the antecedent part of each of the above rule is connected by and operator we use min operator to evaluate strength of each rule.

Strength of Rule 1 DMGM

$$S1 = \min(\mu_{MD}(60), \mu_{MG}(70))$$
$$= \min\left(\frac{4}{5}, \frac{3}{5}\right)$$
$$= \frac{3}{5}$$

Strength of Rule 2 DMGL

$$S2 = \min(\mu_{MD}(60), \mu_{GL}(70))$$
$$= \min\left(\frac{4}{5}, \frac{2}{5}\right)$$
$$= \frac{2}{5}$$

Strength of Rule 3 DLGM

$$S3 = \min(\mu_{LD}(60), \mu_{MG}(70))$$
$$= \min\left(\frac{1}{5}, \frac{3}{5}\right)$$
$$= \frac{1}{5}$$

Strength of Rule 4 DLGL

$$S4 = \min(\mu_{LD}(60), \mu_{LG}(70))$$
$$= \min\left(\frac{1}{5}, \frac{2}{5}\right)$$
$$= \frac{1}{5}$$

Dirt \ Grease		MG	LG
	X	X	X
MD	X	M	L
LD	X	L	VL

Dirt \ Grease		MG	LG
	X	X	X
MD	X	3/5	2/5
LD	X	1/5	1/5

MAX Membership Function

Step05: Defuzzification

Since we use "Mean of Max" defuzzification technique

Maximum strength = Max(S1, S2, S3, S4)

= Max(3/5, 2/5, 1/5, 1/5)

= 3/5

- This corresponds to rule 1
- Rule 1: Dirt is medium and Grease is medium has maximum strength (3/5)
- To find out the final defuzzified value, we now take average (mean) of $\mu_M(z)$.

$$\mu_M(z) = \frac{z - 10}{15}$$

$$\frac{3}{5} = \frac{z - 10}{15}$$

$$\mu_M(z) = \frac{40 - z}{15}$$

$$\frac{3}{5} = \frac{40 - z}{15}$$

Wash Time

$\therefore z = 19$ $\therefore z = 31$

$$\therefore Z = \frac{19 + 31}{2}$$

$$Z = 25\ min$$