# Questions on operations on Fuzzy sets

**Que.** **If A and B are two fuzzy sets with membership functions**

**μA(x) = {0.6, 0.5, 0.1, 0.7, 0.8}**
**μB(x) = {0.9, 0.2, 0.6, 0.8, 0.5}**

**Then the value of μ(A∪B)′(x) will be**

**a.** {0.9, 0.5, 0.6, 0.8, 0.8}

**b.** {0.6, 0.2, 0.1, 0.7, 0.5}

**c.** {0.1, 0.5, 0.4, 0.2, 0.2}

**d.** {0.1, 0.5, 0.4, 0.2, 0.3}

**Answer:** {0.1, 0.5, 0.4, 0.2, 0.2}

**Que.** **If A and B are two fuzzy sets with membership functions:**

**μa(χ) ={0.2,0.5.,0.6,0.1,0.9}**

**μb (χ)= {0.1,0.5,0.2,0.7,0.8}**

**then the value of μa ∩ μb will be**

**a.** {0.2,0.5,0.6,0.7,0.9}

**b.** {0.2, 0.5,0.2, 0.1,0.8}

**c.** {0.1, 0.5, 0.6, 0.1,0.8}

**d.** {0.1, 0.5, 0.2, 0.1,0.8}

**Answer:** {0.1, 0.5, 0.2, 0.1,0.8}

**Que.** **Given U = {1,2,3,4,5,6,7}**

**A = {(3, 0.7), (5, 1), (6, 0.8)}**

**then A will be: (where ~ → complement)**

**a.** {(4, 0.7), (2,1), (1,0.8)}

**b.** {(4, 0.3.): (5, 0), (6. 0.2) }

**c.** {(l, 1), (2, 1), (3, 0.3), (4, 1), (6,0.2), (7, 1)}

**d.** {(3, 0.3), (6.0.2)}

**Answer:** {(l, 1), (2, 1), (3, 0.3), (4, 1), (6,0.2), (7, 1)}

# BASICS of FUZZY SET

## Height of a Fuzzy Set

**Definition**
The *height* $h(\mu)$ of a fuzzy set $\mu \in \mathcal{F}(X)$ is the largest membership grade obtained by any element in that set. Formally,

$$h(\mu) = \sup_{x \in X} \mu(x).$$

$h(\mu)$ may also be viewed as supremum of $\alpha$ for which $[\mu]_\alpha \neq \emptyset$.

**Definition**
A fuzzy set $\mu$ is called *normal* when $h(\mu) = 1$.
It is called *subnormal* when $h(\mu) < 1$.

- **Normal fuzzy set**: A fuzzy set whose membership function has at least one element x in the universe whose membership value is unity.
  - **Prototypical element**: The element for which the membership is equal to 1.
- **Subnormal fuzzy set**: A fuzzy set wherein no membership function has it equal to 1.
- **Convex fuzzy set:** A convex fuzzy set has membership function whose membership values are strictly monotonically increasing or strictly monotonically decreasing or strictly monotonically increasing than strictly monotonically decreasing with increasing values for the elements in the universe.
- **Nonconvex fuzzy set:** the membership value of the membership function is not strictly monotonically increasing or decreasing or strictly monotonically increasing than decreasing.

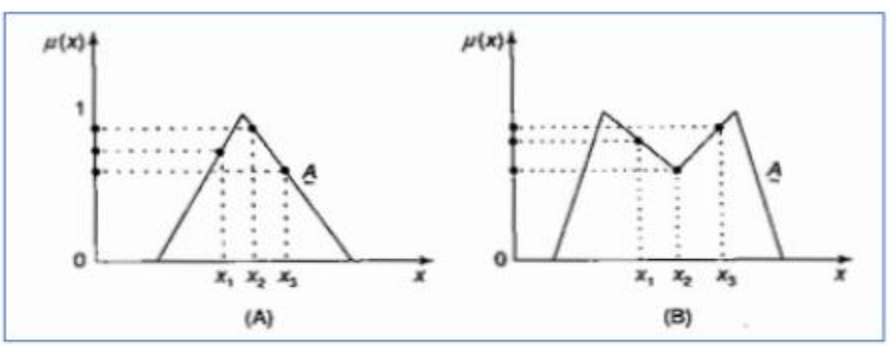



**Figure 8.3: (A) Convex Normal Fuzzy Set and (B) Nonconvex Normal Fuzzy Set**

The intersection of two convex fuzzy set is also a convex fuzzy set. The element in the universe for which a particular fuzzy set A has its value equal to 0.5 is called **crossover point** of membership function. There can be more than one crossover point in fuzzy set. The maximum value of the membership function of the fuzzy set A is called **height of the fuzzy set**. If the height of the fuzzy set is less than 1, then the fuzzy set is called **subnormal fuzzy set**. When the fuzzy set A is a convex single –point normal fuzzy set defined on the real time, then A is termed as a **fuzzy number**.

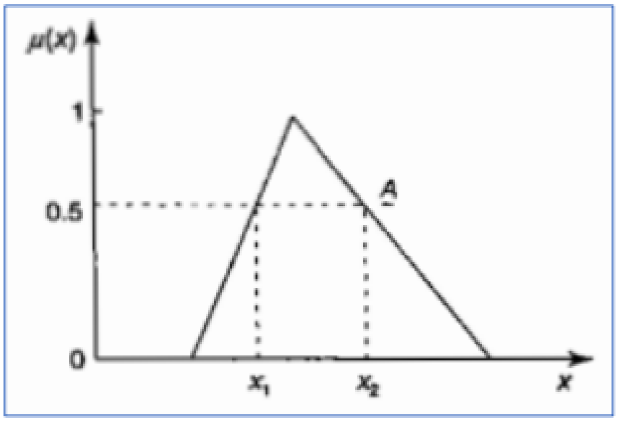



**Figure 8.4: Crossover Point of a Fuzzy Set**

**Definition 2.2**: Given a fuzzy set $A$ in $X$ and any real number $\alpha \in [0, 1]$, then the $\alpha$ -cut or $\alpha$ -level or cut worthy set of $A$, denoted by $^{\alpha}A$ is the crisp set

$$^{\alpha}A = \{x \in X: \ \mu_A(x) \geq \alpha\}$$

The strong $a$ cut, denoted by $^{\alpha+}A$ is the crisp set

$$^{\alpha+}A = \{x \in X: \ \mu_A(x) > \alpha\}$$

1. Consider a fuzzy set A defined on the interval X = [0, 10] of integers by the membership Junction
μA(x) = x / (x+2)
Then the α cut corresponding to α = 0.5 will be

a. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

b. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

c. {2, 3, 4, 5, 6, 7, 8, 9, 10} Answer

d. None of the above

Explanation:-
In the fundamentals, refer to the answer given for question no. 6 regarding α-cut.
α-cut of a fuzzy set A denoted as Aα, is the crisp set comprised of the elements x of a universe of discourse X for which the membership function of A is greater than or equal to α.
Given, x = In the range [0,10]
Membership function = x/x+2
Calculate the value of membership function for the interval from 0 to 10, substituting in the formula x/x+2.
μA(0) = 0 / 0+ 2 = 0

μA(1) = 1 / 1+ 2 = 0.33

μA(2) = 2 / 2+ 2 = 0.5

μA(3) = 3 / 3+ 2 = 0.6

μA(4) = 4 / 4+ 2 = 0.66

μA(5) = 5 / 5+ 2 = 0.71

μA(6) = 6 / 6+ 2 = 0.75

μA(7) = 7 / 7+ 2 = 0.77

μA(8) = 8 / 8+ 2 = 0.8

μA(9) = 9 / 9+ 2 = 0.81

μA(10) = 10 / 10+ 2 = 0.83

α= 0.5. We have to find the corresponding α-cut,

That will be a crisp set, having those values of x, for which the membership function is returning a value of 0.5 or above.

μA(2) = 0.5 and all the values of x above 2 is getting a value greater than 0.5. So the crisp set will contain the following values.

{ 2,3,4,5,6,7,8,9,10}.

So the correct answer is C.

**7.** Consider a fuzzy set old as defined below

Old = {(20, 0.1), (30, 0.2), (40, 0.4), (50, 0.6), (60, 0.8), (70, 1), (80, 1)}

Then the alpha-cut for alpha = 0.4 for the set old will be

**a.** {(40,0.4)}

**b.** {50, 60, 70, 80}

**c.** {(20, 0.1), (30, 0.2)}

**d.** {(20, 0), (30, 0), (40, 1), (50,1), (60, 1), (70, 1), (80, 1)} **Answer**

29. The height h(A) of a fuzzy set A is defined as h(A) =sup A(x) where x belongs to A. Then the fuzzy set A is called normal when

(A)h(A)=0

(B)h(A)<0

(C)h(A)=1

(D)h(A)<1

Ans:- C

**7. A fuzzy set whose membership function has at least one element x in the universe whose membership value is unity is called**

A sub normal fuzzy sets

B normal fuzzy set

C convex fuzzy set

D concave fuzzy set

Answer Comment (8) (2)

Option : **B**

## 9. A fuzzy set wherein no membership function has its value equal to 1 is called

A normal fuzzy set

B subnormal fuzzy set.

C convex fuzzy set

D concave fuzzy set

Answer Comment (2) (0)

Option : **B**

19. There are also other operators, more linguistic in nature, called _______ that can be applied to fuzzy set theory.

a) Hedges

b) Lingual Variable

c) Fuzz Variable

d) None of the mentioned

View Answer Answer: a

29. In a Fuzzy set a prototypical element has a value

a) 1

b) 0

c) infinite

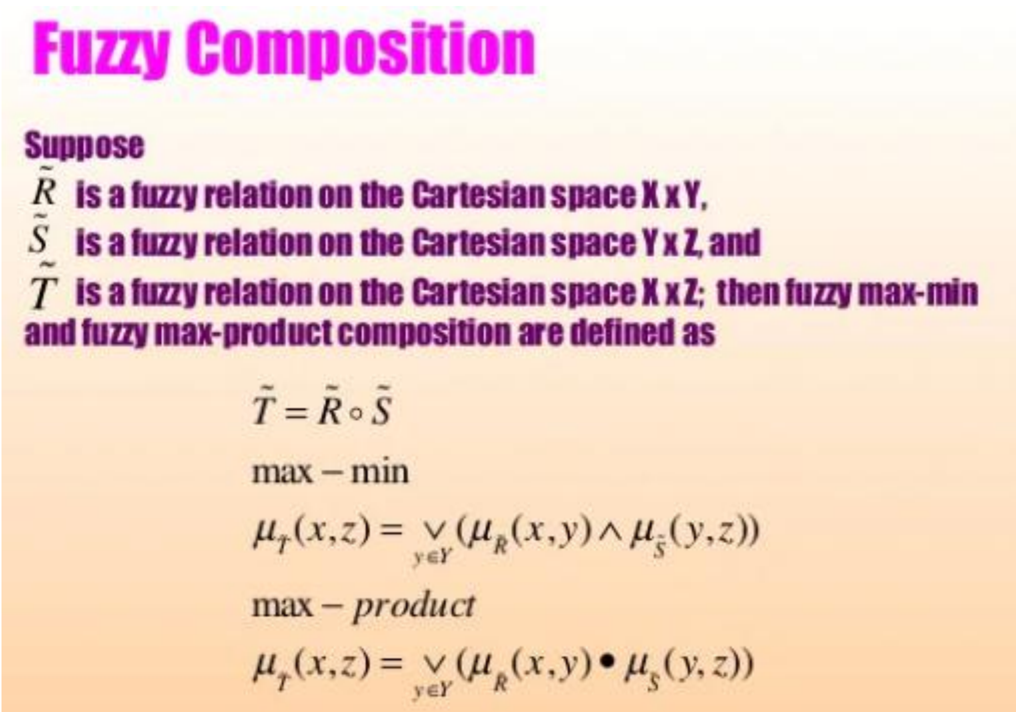
d) not defined

Ans: A

## Fuzzy Composition

There are two types of fuzzy composition techniques:

1. Fuzzy Max-min composition 2. Fuzzy Max-product composition



## Fuzzy Composition: Example (max-min)

$$X=\{x_1,x_2\},\quad Y=\{y_1,y_2\},\text{ and }\quad Z=\{z_1,z_2,z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R}=\begin{matrix} & y_1 & y_2 \\ x_1 & 0.7 & 0.5 \\ x_2 & 0.8 & 0.4 \end{matrix} \quad\text{and}\quad \tilde{S}=\begin{matrix} & z_1 & z_2 & z_3 \\ y_1 & 0.9 & 0.6 & 0.5 \\ y_2 & 0.1 & 0.7 & 0.5 \end{matrix}$$

Using max-min composition,

$$\mu_{\tilde{T}}(x_1,z_1)=\bigvee_{y\in Y}(\mu_{\tilde{R}}(x_1,y)\wedge\mu_{\tilde{S}}(y,z_1))$$
$$=\max[\min(0.7,0.9),\min(0.5,0.1)]$$
$$=0.7$$

$$\tilde{T}=\begin{matrix} & z_1 & z_2 & z_3 \\ x_1 & 0.7 & 0.6 & 0.5 \\ x_2 & 0.8 & 0.6 & 0.5 \end{matrix}$$

## Fuzzy Composition: Example (max-Prod)

$$X=\{x_1,x_2\},\quad Y=\{y_1,y_2\},\text{ and }\quad Z=\{z_1,z_2,z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R}=\begin{matrix} & y_1 & y_2 \\ x_1 & 0.7 & 0.5 \\ x_2 & 0.8 & 0.4 \end{matrix} \quad\text{and}\quad \tilde{S}=\begin{matrix} & z_1 & z_2 & z_3 \\ y_1 & 0.9 & 0.6 & 0.5 \\ y_2 & 0.1 & 0.7 & 0.5 \end{matrix}$$

Using max-product composition,

$$\mu_{\tilde{T}}(x_2,z_2)=\bigvee_{y\in Y}(\mu_{\tilde{R}}(x_2,y)\bullet\mu_{\tilde{S}}(y,z_2))$$
$$=\max[(0.8,0.6),(0.4,0.7)]$$
$$=0.48$$

$$\tilde{T}=\begin{matrix} & z_1 & z_2 & z_3 \\ x_1 & .63 & .42 & .35 \\ x_2 & .72 & .48 & .40 \end{matrix}$$

**Example:**

X = {$x_1$, $x_2$}, Y = {$y_1$, $y_2$}, and Z = {$z_1$, $z_2$, $z_3$}. Consider the following fuzzy relations:

$$\bar{R}=\begin{matrix} & y_1 & y_2 \\ x_1 & 0.7 & 0.6 \\ x_2 & 0.8 & 0.3 \end{matrix}$$

Relation R

$$\bar{S}=\begin{matrix} & z_1 & z_2 & z_3 \\ y_1 & 0.8 & 0.5 & 0.4 \\ y_2 & 0.1 & 0.6 & 0.7 \end{matrix}$$

Relation S

**Solution:**

So ultimately, we have to find the elements of matrix,

$$\bar{T} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \left[ \begin{array}{ccc} \quad & \quad & \quad \\ \quad & \quad & \quad \end{array} \right] \end{array}$$

Composition of relation $\underline{R}$ and $\underline{S}$

## Max-Min Composition:

Max-min composition is defined as,

$\underline{T} = \underline{R} \circ \underline{S} = \boldsymbol{\mu}_{\underline{T}}(x, z) = \bigvee_{y \in Y} ( \boldsymbol{\mu}_{\underline{R}}(x, y) \vee \boldsymbol{\mu}_{\underline{S}}(y, z) )$

From the given relations $\bar{R}$ and $\bar{S}$,

$\boldsymbol{\mu}_{\underline{T}}(x_1, z_1) = \max ( \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_1), \boldsymbol{\mu}_{\underline{S}}(y_1, z_1)), \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_2), \boldsymbol{\mu}_{\underline{S}}(y_2, z_1)) )$

$= \max(\min(0.7, 0.8), \min(0.6, 0.1)) = \max(0.7, 0.1) = 0.7$

$\boldsymbol{\mu}_{\underline{T}}(x_1, z_2) = \max ( \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_1), \boldsymbol{\mu}_{\underline{S}}(y_1, z_2)), \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_2), \boldsymbol{\mu}_{\underline{S}}(y_2, z_2)) )$

$= \max(\min(0.7, 0.5), \min(0.6, 0.6)) = \max(0.5, 0.6) = 0.6$

$\boldsymbol{\mu}_{\underline{T}}(x_1, z_3) = \max ( \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_1), \boldsymbol{\mu}_{\underline{S}}(y_1, z_3)), \min( \boldsymbol{\mu}_{\underline{R}}(x_1, y_2), \boldsymbol{\mu}_{\underline{S}}(y_2, z_3)) )$

$= \max(\min(0.7, 0.4), \min(0.6, 0.7)) = \max(0.4, 0.6) = 0.6$

$\boldsymbol{\mu}_{\underline{T}}(x_2, z_1) = \max ( \min( \boldsymbol{\mu}_{\underline{R}}(x_2, y_1), \boldsymbol{\mu}_{\underline{S}}(y_1, z_1)), \min( \boldsymbol{\mu}_{\underline{R}}(x_2, y_2), \boldsymbol{\mu}_{\underline{S}}(y_2, z_1)) )$

$= \max(\min(0.8, 0.8), \min(0.3, 0.1)) = \max(0.8, 0.1) = 0.8$

$\boldsymbol{\mu}_{\underline{T}}(x_2, z_2) = \max ( \min( \boldsymbol{\mu}_{\underline{R}}(x_2, y_1), \boldsymbol{\mu}_{\underline{S}}(y_1, z_2)), \min( \boldsymbol{\mu}_{\underline{R}}(x_2, y_2), \boldsymbol{\mu}_{\underline{S}}(y_2, z_2)) )$

$= \max(\min(0.8, 0.5), \min(0.3, 0.6)) = \max(0.5, 0.3) = 0.5$

$\mu_{\underline{T}}(x_2, z_3)$ = max ( min( $\mu_{\underline{R}}(x_2, y_1)$, $\mu_{\underline{S}}(y_1, z_3)$), min( $\mu_{\underline{R}}(x_2, y_2)$, $\mu_{\underline{S}}(y_2, z_3)$) )

= max(min(0.8, 0.4), min(0.3, 0.7)) = max(0.4, 0.3) = 0.4

$$\bar{T} = \begin{matrix} & z_1 & z_2 & z_3 \\ x_1 & 0.7 & 0.6 & 0.6 \\ x_2 & 0.8 & 0.5 & 0.4 \end{matrix}$$

Max-Min composition of Relations

### Max-Product Composition:

$\underline{T} = \underline{R} \circ \underline{S} = \mu_{\underline{T}}(x, z) = \bigvee_{y \in Y} ( \mu_{\underline{R}}(x, y) \cdot \mu_{\underline{S}}(y, z) )$

$= \max_{y \in Y} ( ( \mu_{\underline{R}}(x, y) \cdot \mu_{\underline{S}}(y, z) ) )$

$\mu_{\underline{T}}(x_1, z_1)$ = max ( ($\mu_{\underline{R}}(x_1, y_1) \times \mu_{\underline{S}}(y_1, z_1)$), ( $\mu_{\underline{R}}(x_1, y_2) \times \mu_{\underline{S}}(y_2, z_1)$) )

= max((0.7 × 0.8), (0.6 × 0.1)) = max(0.56, 0.06) = 0.56

$\mu_{\underline{T}}(x_1, z_2)$ = max ( ( $\mu_{\underline{R}}(x_1, y_1) \times \mu_{\underline{S}}(y_1, z_2)$), ( $\mu_{\underline{R}}(x_1, y_2) \times \mu_{\underline{S}}(y_2, z_2)$) )

= max( (0.7 × 0.5), (0.6 × 0.6)) = max(0.35, 0.36) = 0.36

$\mu_{\underline{T}}(x_1, z_3)$ = max ( ( $\mu_{\underline{R}}(x_1, y_1) \times \mu_{\underline{S}}(y_1, z_3)$), ( $\mu_{\underline{R}}(x_1, y_2) \times \mu_{\underline{S}}(y_2, z_3)$) )

= max((0.7 × 0.4), (0.6 × 0.7)) = max(0.28, 0.42) = 0.42

$\mu_{\underline{T}}(x_2, z_1)$ = max ( ( $\mu_{\underline{R}}(x_2, y_1) \times \mu_{\underline{S}}(y_1, z_1)$), ( $\mu_{\underline{R}}(x_2, y_2) \times \mu_{\underline{S}}(y_2, z_1)$) )

= max((0.8 × 0.8), min(0.3 × 0.1)) = max(0.64, 0.03) = 0.64

$\mu_{\underline{T}}(x_2, z_2) = \max ( ( \mu_{\underline{R}}(x_2, y_1) \times \mu_{\underline{S}}(y_1, z_2)), ( \mu_{\underline{R}}(x_2, y_2) \times \mu_{\underline{S}}(y_2, z_2)) )$

$= \max((0.8 \times 0.5), (0.3 \times 0.6)) = \max(0.4, 0.18) = 0.40$

$\mu_{\underline{T}}(x_2, z_3) = \max ( ( \mu_{\underline{R}}(x_2, y_1) \times \mu_{\underline{S}}(y_1, z_3)), ( \mu_{\underline{R}}(x_2, y_2) \times \mu_{\underline{S}}(y_2, z_3)) )$

$= \max((0.8 \times 0.4), (0.3 \times 0.7)) = \max(0.32, 0.21) = 0.32$

$$\bar{T} = \begin{matrix} & z_1 & z_2 & z_3 \\ x_1 & 0.56 & 0.36 & 0.42 \\ x_2 & 0.64 & 0.40 & 0.32 \end{matrix}$$

Max-Product composition